NAZARİYAT

Taqī al-Dīn al-Rāṣid and His Treatise on Algebra

Mathematical Evaluation, Translation, and *Editio Princeps**

Elif Baga**

Abstract: Taqī al-Dīn al-Rāṣid is one of the most important representatives of the Ottoman tradition on mathematical sciences. His research focus being on astronomy, astronomical instruments, mathematics, optics, mechanics, and physics is understood from his surviving works. Taqī al-Dīn's establishment and management of the Istanbul Observatory, which was the first observatory in the Ottoman Empire, made him an important figure in many ways. However, despite the aforementioned importance, the mathematics he learned, taught, produced and used has been a subject for very few studies. The primary way of determining the quality and the level of a work is to look at the kind of tools used for its creation. Therefore, this article will present the edition princeps, the translation and the evaluation of Taqī al-Dīn al-Rāṣid's treatise on algebra, *al-Nisab al-mutashākila fī ʿilm al-jabr wa-l-muqābala*. It will be presented in the context of the idea that revealing a scientific character and the career of scholars who stand out in mathematical sciences can be possible by analyzing their mathematical works. The nature of the science of algebra, which can be applied to any problem encountered on any subject regardless of geometry or arithmetic, makes this idea more meaningful. For the correct examination of classical mathematical works, first the original text is verified and transformed into a format that provides an easy reading, then it is to be translated into the desired language. Finally a mathematical analysis and historical evaluation are needed to explain the main structure and justification of the content of the article.

Keywords: Taqī al-Dīn al-Rāṣid, history of algebra in Ottomans, history of mathematics in Ottomans, algebra, mathematics.

* I would like to thank Dr. Taha Yasin Arslan for reading and evaluating the English version of this article.

** Assistant Professor, Department of the History of Science, Istanbul Medeniyet University. correspondence: elifbaga@gmail.com

DOI dx.doi.org/10.12658/Nazariyat.7.2.M0151en

iD https://orcid.org/0000-0003-2951-2563

Received 29 March 2021 Accepted 2 May 2021

Atıf© Baga, Elif. "Taqī al-Dīn al-Rāṣid and His Treatise on Algebra: Mathematical Evaluation, Translation, and *Editio Princeps*", *Nazariyat* 7/2 (November 2021): 1-54.

1. Introduction

The mathematical works of a scientist who did research and wrote works in the fields of mathematics, physics, optics, mechanics, and astronomy can provide vital data on issues such as the methods of the knowledge in these fields, the ways knowledge was constructed and its points of view and levels. Because mathematics is a basic science structurally prone to being a tool, one's knowledge and experience of and talent with it determines their position in every field they use it.

Taqī al-Dīn al-Rāṣid was the founder of the first Ottoman observatory, the author of a skilled book on mechanics in his field, the author of the most comprehensive Ottoman work on optics and the compiler of the very-first *zîj* that used the decimal system. Therefore, identifying and analyzing the mathematics he used will obviously widen the horizon on making sense of its scientific character. Although how many mathematics books the author wrote during his lifetime is not exactly known, one calculus, one algebra and four theoretical geometry (*al-handasa*) works have survived. His work on calculations (*al-ḥisāb*) is titled as *Bughyat al-ṭullāb fī ʻilm al-ḥisāb*[1] and his treatise on algebra as *al-Nisab al-mutashākila fī ʻilm al-jabr wa-l-muqābala*. With the exception of *Tastīh al-ukar*[2], each work on theoretical geometry involves a specific problem consisting of a few pages.[3] Of course, the fact that these works contain one or more of the reasons for writing such as being a textbook, putting forward a new theory, adding to an existing theory and showing a different field of application may change the level of understanding of Taqī al-Dīn al-Rāṣid's scientific character. However, in any case, publishing these mathematical works, translating them into English, analyzing them mathematically and evaluating them historically, in short, putting them at the disposal of science historians is

1 For manuscript copies, see. Zeytinoğlu İlçe Halk Kütüphanesi, Zeytinoğlu Koleksiyonu, 43 Ze 303/1, f. 44+16; Süleymaniye Kütüphanesi, Carullah 1454, f. 56; https://digi.vatlib.it/mss/detail/Sbath.496

2 A copy of the work, which roughly describes the transfer of the spherical object to the plane, is between the folios of Kandilli Observatory 415/5, 80v-92r. For other copies, see: Ramazan Şeşen vd., *Osmanlı Astronomi Literatürü Tarihi (OALT)* (İstanbul: IRCICA, 1997), I, 205.

3 Apart from the ones mentioned above, the three mathematical works whose copies have survived are as follows: *Jawāb suāl an muthallath*… on the determination of the angles of a triangle whose sides are known but not perpendicular, *Risāla fī tahqīq mā qālahu Allāma Jamshīd Kāshī*… on the verification of al-Kashi's statement about the ratio between diameter and circumference and the *Risāla fī a'māl al-mīzān al-tabī*, which talks about Archimedes' scales, can be counted. For more information see: Ramazan Şeşen vd., *Osmanlı Matematik Literatürü Tarihi* (*OMLT)* (İstanbul: IRCICA, 1999), I, 84-6. The trigonometry section in the astronomy work *Sidrat al-muntaha* can also be added to the mathematical works. For a comparative analysis of this passage with the work of Copernicus, see Sevim Tekeli, "Trigonometry in the Sixteenth Century: Copernicus and Taqī-Dīn", *Erdem* 2/4 (1986): 247-72.

essential for Taqī al-Dīn al-Rāṣid's studies on a small scale. It's also vital for the history of science on a large scale to progress in the right direction.

Taqī al-Dīn's work *Bughyat al-ṭullāb* is divided into three parts: calculating decimals, calculating sexagesimals (60-based number system), and algebra. In terms of its volume, the work exceeds the limits of one single article. As the name suggests, it was written at the request of students. The work has not been the subject of any research until now and it may have been written for Taqī al-Dīn's students while he was a professor in the madrasa or for his students at the Istanbul Observatory when he was the Chief Astronomer/Astrologer of the Ottoman Court (Müneccimbaşı).

Another surviving work from the author is his treatise on algebra, which can be described as a concise volume. Algebra first appeared and developed in the mathematical tradition of Islamic civilization as a branch of science and has been used by the scholars of this tradition to solve multivariate and complex problems from al-farāiḍ and transactions to applied geometry and astronomy, just like a key for opening difficult locks. Assuming that Taqī al-Dīn al-Rāṣid also used the science of algebra as the predecessors in his tradition had, for him to compose such a treatise would be considered normal because of his interest in many branches of the astronomical sciences such as optics and mechanics. He may even have written the treatise on algebra for his colleagues for being used in the fields that require group work, such as mechanics and astronomy. Whether this possibility is true or not, the author's ideas on algebra make presenting an analysis and evaluation of his work important in terms of giving an idea about his style and how he coped with mathematical problems in other fields. Due to this aforementioned importance, the current article will present the introduction to *al-Nisab al-mutashākila fī 'ilm al-jabr wa-al-muqābala* as well as its historical evaluation, mathematical analysis, full English translation and *editio princeps*.

Although Taqī al-Dīn al-Rāṣid is already well known, the article will begin by giving his biography, albeit in brief. This is because the development of his scientific career is directly related to his written works and also because some controversial situations related to his life had occurred. Immediately after this, the article will present detailed introductory information so that the content of the work can be viewed holistically, followed by the mathematical analysis and historical evaluation of the work. The first purpose of this section is to show the content of the work making use of modern mathematical symbols and thus bring mathematics that were verbally expressed about five centuries ago to the minds of today's mathematicians or historians of mathematics. The other purpose is to show the historical context of algebraic expressions and operations and to establish their relationship with the classical algebraic tradition to be able to understand the position this work has among these. As this process will be done by giving footnotes to expressions and operations during mathematical analysis, it has been titled under "Mathematical Analysis and Historical Evaluation". Last to be presented will be the complete English translation of the text and its *editio princeps*.

As far as can be determined, the first study on Taqī al-Dīn's work was done by Melek Dosay Gökdoğan in 1997.[4] Her article, *Takiyüddin'in Cebir Risalesi* [Taqī al-Dīn's Treatise on Algebra] begins with a very short biography and continues with the section "Takiyüddin'in Cebiri" [Taqī al-Dīn's Algebra] describing the content of the work. According to the conveyed information, the article then goes into a free-style translation of the text into Turkish accompanied by the handwritten text from the Oxford I. 881/3 copy minus the *editio princeps*. The shortcomings of the nearly quarter-century-old article can be considered natural, considering both the conditions of the period and the absence of any previous study on the work.

In 2003, six years after Dosay's publication, Moustafa Mawaldi published *Nisab al-mutashākila* in the journal *Abhath al-mu'tamar al-sanavi li-tarīkh al-ʿulum ʿinda al-ʿArab*.[5] According to the information he gave, Mawaldi only used the Oxford I.881/3 copy and added an evaluation section to his work.

Despite the two existing studies, the reasons for reintroducing and examining the work under the previously mentioned titles can be listed as follows:

(a) Even if a few minor errors in the already published text of the work are disregarded, (i) the punctuation, headings and the paragraphing method used in the publication are not suitable for classical mathematical works[6] and (ii) the publication in which the text is published is not accessible to the majority of readers.

4 Melek Dosay, "Takîyüddîn'in Cebir Risalesi", *Belleten* 61 (1997): 301-20.

5 Moustafa Mawaldi, "Tahqiq wa dirasah makhtut *Ketab Nisab al-mutashākilah fī ʿilm al-jabr wa-al-muqābalah* li-Taqī al-Dīn b. Maʿrūf", *Abhath al-mu'tamar al-sanavi li-tarikh al-ulum inda al-arab* (Aleppo: Ma'had al-Turath al-İlmi al-Arabî, 2003), 445-70.

6 No methodology research has been carried out in Turkey so far for the *taḥqīq*, Turkish translation, mathematical analysis and evaluation (*dirāsa*) of the works in the field of classical mathematical sciences. Based on the existing ISAM *taḥqīq* and *dirāsa* principles, new methods and techniques should be determined on how to work on the manuscripts of the matehamatical sciences in the most accurate and most effective way. In this respect, we are working to propose a methodology for the examination of works in the field of mathematical sciences such as calculation (*al-ḥisāb*), algebra, applied geometry (*al-masāḥa*) and theoretical geometry (*al-handasa*), and it is planned to be presented to the scientific public this year.

(b) When the name and number of the collection was changed in the Oxford copy, which is the only accessible copy of *Nisab al-mutashākila*, is not known. However, all publications mentioning the work contain old information in the form of Oxford I.881/3 and this information should be updated to MS Greaves 3/3.

(c) A concise and complete Turkish translation is required in place of the previously published free translation.

(d) The need exists to translate the entire text into the language of mathematics (i.e., represent the text with mathematical notations) so that the work becomes available to all who are familiar with modern mathematics in its entirety.

(e) Revealing the continuity of the issues and expressions that stand out is needed in terms of their historical background in the Islamic and Ottoman periods through footnotes while giving mathematical representations, thus indicating the position of Taqī al-Dīn's algebraic work in terms of the history of mathematics.

2. The Life of Taqī al-Dīn al-Rāṣid

Abū Bakr Taqī al-Dīn Muḥammad ibn Zayn al-Dīn ibn Maʿrūf ibn Aḥmad al-Rāṣid al-Dimashqī was born in the holy land on Ramadan 4, 932 (June 14, 1526) according to his own words[7] and spent a significant part of his life in Damascus due to his father's work in the Sibāiyya and Taqawiyya madrasas of Damascus.[8] As a requirement of the tradition he was in, he was taught the Qur'an, Arabic, hadith, fiqh and tafsir sciences, also mathematics and astronomy from various teachers in the madrasas of Damascus and Egypt. He took lessons from Muḥammad ibn Abī al-Fatḥ al-Ṣūfī[9] (d. 950/1543) in the field of astronomy and from Shihāb al-Dīn al-Ghazzī al-Shāfiʻī[10] (d. 983/1576) in mathematics. Under the influence of his

7 Taqī al-Dīn Rāṣid, *Sidrat al-muntaha*, Kandilli Rasathanesi 208, f. 1v ve Nuruosmaniye Kütüphanesi 2930, f. 1v.

8 Although some researchers describe him as of Arab origin due to his birth and growth in Arab lands, a few studies based on the chain of names that Rāṣid mentioned in his works reveal that he is Turkish. For more detailed information on this subject, see: Ramazan Şeşen, "Meşhur Osmanlı Astronomu Takıyyüddin Râsıd'ın Soyu Üzerine", *Erdem* 4/10 (1988): 165-172.

9 For more information about Ebū al-Fath al-Sūfī, who was one of the mathematician-astronomer representatives of the Egyptian School, who lived between the end of the 9/15. century and the beginning of the 10/16. century, and his works, see: İhsan Fazlıoğlu, "Ibn Abī al-Fatḥ al-Ṣūfī: Shams al-Dīn Abū 'Abd Allāh Muḥammad ibn Abī al-Fatḥ al-Ṣūfī", *The Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey vd. (Newyork: Springer, 2007), 547; Şeşen vd., *OMLT*, I, 59-62; Şeşen vd., *OALT*, I, 130.

10 For Ghazi, who was born in Damascus, completed his education in Egypt, and returned to his hometown, he worked as a professor there, see: Şeşen vd., *OMLT*, I, 79-80.

intense interest in mathematical sciences, he quickly trained himself in these. He was in Istanbul with his father Maʿrūf Efendī between 1550-1555 and benefited from the scientific knowledge of Çivizāde Hacı Mehmet Efendī,[11] Ebū al-Suʿūd Efendī, Qutb al-Dīn-zāda Muḥammad Efendī[12] and Saçlı Emir Efendī.[13] After this date, he returned to Egypt and taught at the Shay'khūniyya and Sarghatmishiyya madrasas in Cairo for a while, then came to Istanbul for the second time and was appointed as a *mudarris* to the Edirnekapı Madrasa during Samiz Ali Pasha's grand viziership (1561-1565). However, because of the difficulties of being away from his family, he left this job and returned to Egypt where he served as both a *mudarris* and qadi before the death of Süleyman the Magnificent in 1566. During the reign of Salim II (1566-1574), who succeeded Süleyman the Magnificent, he was deputized for Çivizāde and Nişancızāde, who were performing the duties of the Qādī of Egypt. When Kazasker ʿAbd al-Karīm Efendī was appointed as the Qādī of Egypt after Nişancızāde, Taqī al-Dīn al-Rāṣid received much attention from this new *qādī* and his grandfather[14], Qutb al-Dīn, in terms of supporting and encouraging his scientific studies, especially his research on mathematics and astronomy. From the influence of his great-grandfather Ali Kuşçu's great success in mathematics and astronomy, ʿAbd al-Karīm Efendī handed the works and equipment of Kuşçu, Jamshīd Kāshī, and Qādī-zāda al-Rūmī over to Taqī al-Dīn al-Rāṣid. Thus, this continued to support Taqī al-Dīn's developments in these sciences, who devoted

11 Çivizāde Mehmet Efendī, son of Sheikh al-Islam Çivizāde Mohy al-Dīn Mehmet Efendī (896/1491-954/1547) who also served as a Sheikh al-Islam, was born in Istanbul (937/1530-995/1587). After taking lessons from various teachers, he served as a mudarris in many madrasahs and as a judge in many cities. For more information about Çivizāde Mehmet Efendī, his father and the Çivizāde family, see: Mehmet İpşirli, "Çivizâdeler", *DİA*, VIII, 349-50.

12 Mullah Muḥammad Chalabī b. Qutb al-Dīn, after working as a mudarris in various madrasas, worked as a judge in Adirna and Istanbul and as an Anatolian Kazasker in turn. He died in 957/1551. For more information. Taşköprülüzâde, *Osmanlı Bilginleri*, trc. Muharrem Tan (İstanbul: İz Yayıncılık, 2007), 323.

13 His real name is Molla Mohy al-Dīn Mehmet ibn ʿAbd al-Awwal al-Tabrīzī. His father was a Tabriz judge under the rule of Aq-qoyun, who belonged to the Hanafi sect. Probably for this reason, it came under the auspices of the Ottoman Empire after the establishment of the Safavid Empire. Sultan II. He served as a judge in various regions during the reigns of Bayezid and Süleyman the Magnificent. For more information. Taşköprülüzâde, *Osmanlı Bilginleri*, 346.

14 Although there are phrases like "Qutb al-Dīn, the father of Kazasker ʿAbd al-Karīm Efendī" in all the sources giving the biography of Taqī al-Dīn, the results of Mehmet Arıkan, who has been researching Qutb al-Dīn Chalabī for a long time and preparing an article on this subject, revealed that ʿAbd al-Karīm Efendī is the grandson of Qutb al-Dīn Chalabī. In addition, since ʿAbd al-Karīm Mehmet is shown as the son of Qutb al-Dīn's son Mehmet in the genealogy given by Süheyl Ünver, the term "grandfather" was considered more appropriate. See: Mehmet Arikan, "Kadızâde-i Rûmî", *Temel İslam Ansiklopedisi*, ed. Tuncay Başoğlu (Ankara: Türkiye Diyanet Vakfı Yayınları, 2020), 538; Süheyl Ünver, *İstanbul Risaleleri*, haz. İsmail Kara (İstanbul: İBB Yayınları, 1995), II, 283-4.

himself to mathematics and astronomy in response to this interest. He continued his observations and research even while working as a judge in Egypt and Palestine, maintaining and producing new studies. He came to Istanbul for the last time in 978/1570, during the reign of Salim II. He came under the patronage of Hodja Saʿd al- Dīn Efendī (d. 1007/1599), one of the leading professors of the period and teacher to Prince Murad III and they established a close relationship. It can be said that this behavior of Taqī al-Dīn might have been effective in bringing him to this post after the death of Mustafa ibn Ali al-Muwaqqit[15] (d. 979/1571), the Chief Astronomer/Astrologer of the Ottoman Court (Müneccimbaşı) at the time. Taqī al-Dīn, who continued his observations and research that he'd started in Egypt while performing his job as Chief Astronomer/Astrologer of the Ottoman Court,[16] was presented to Sultan Murad III, attracting the attention of Hodja Saʿd al- Dīn Efendī and the grand vizier of the time, Sokullu Mehmet Pasha, with his astronomy and mathematical studies. *Zīj-i Ulugh Beg*, the product of Samarkand Observatory was the most recent *zīj* in use; under the protection of these individuals. Taqī al-Dīn presented Sultan Murad III with a petition stating his idea that this *zīj* had errors in it and was no longer able to meet current needs. Hence an observatory would be needed to reorganize this *zīj*. The project was approved by the Divan in 1575 and completed in 1577 with the construction of the Istanbul Observatory on the hills of Tophane and its instruments. Observations and research were started next under the presidency of the first and last director of the Istanbul Observatory, Taqī al-Dīn al-Rāṣid. In the same year, Taqī al-Dīn presented Sultan Murad III with a report on some of the predictions he had arrived at as a result of his observations. This report heralded that negative developments would not occur and that the Ottoman army would be successful against the Iranian-Safavid army. Despite the success of the Ottoman army, a series of negative events such as an epidemic of the plague and the successive deaths of some important individuals overshadowed Taqī al-Dīn's scientific reliability in the eyes of the Sultan, which became a link in a chain of reasons that shortened the life of the observatory. Taqī al-Dīn al-Rāṣid was under the auspices of Hodja Saʿd al-Dīn Efendī who in turn had some political

15 One of the important names of 16th century Ottoman astronomy, on the life and works of Mustafa ibn Ali al-Muwaqqit, see Şeşen vd., *OALT*, I, 161-79; İhsan Fazlıoğlu, "Ali al-Muwaqqit: Muslih al-Din Mustafa ibn Ali al-Qustantini al-Rumi al-Hanafi al-Muwaqqit", *The Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey vd. (Newyork: Springer, 2007), 33-4; İhsan Fazlıoğlu, "Mustafa İbn Ali el-Muvakkit", *DİA*, XXXI, 287-8.

16 Regarding the organization of Chief Astronomer/Astrologer, which emerged in the Ottoman state in a regular and systematic way for the first time in history, see. Salim Aydüz, "Osmanlı Devleti'nde Müneccimbaşılık", *Osmanlı Bilimi Araştırmaları* 1 (1995), 159-208.

conflicts that can be said to have been influential in Qādī-zāda Ahmad Shams al-Dīn Efendī, the Shay'kh al-Islam of the time. As a result Qādī-zāda Ahmad Shams al-Dīn Efendī issued a fatwa for the destruction of the Istanbul Observatory, cited the fate of the states that owned the observatory. Ultimately, the first observatory of the Ottoman Empire was destroyed on Thursday, January 22, 1580 per the edict Sultan Murad III gave to Chief Admiral Kılıç Ali Pasha. Taqī al-Dīn, whose observations and research were left unfinished, tried to finish writing his books at home. However, he did not survive long after the destruction of the observatory, died in Istanbul or Damascus in 1585.[17]

Although the Istanbul Observatory[18] was active only for a short time, it was Taqī al-Dīn al-Rāṣid's most important achievement in terms of both the observational instruments he invented and used there for the first time as well as the variety of more useful solutions he brought to astronomical problems. In particular, his efforts in attempting to apply decimal fraction calculus, which his predecessor Jamshīd Kāshī had carried forward, to astronomical calculations for the first time in history are noteworthy, especially in regard to trigonometry, which is a branch of mathematics that had emerged from astronomy. This is because the attempt to replace the use of the sexagesimal calculus system in astronomy, which had been the tradition throughout history, for a new form that was considered more functional was an important step.[19] In order for these attempts and efforts not to be in vain and his research to be accepted, Taqī al-Dīn first discussed the computational systems and the various advantages and difficulties these systems had when applied to various fields. In other words, after he convincingly put forward the justifications for what he was trying to do, he put them into practice.

17 Aydın Sayılı, *The Observatory in Islam* (Ankara: TTK Yayınları, 1988), 289-92; İhsan Fazlıoğlu, "Taqī al-Dīn Abū Bakr Muhammad İbn Zayn al-Dīn Ma'rūf al-Dimashqī al-Hanafī", *The Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey vd. (Newyork: Springer, 2007), 1122-3; Hüseyin Gazi Topdemir, "Takıyyüddin er-Râsıd", *DİA*, XXXIX, 454-5.

18 For detailed information about the establishment of this observatory, its physical structure, activities, the working astronomers, the instruments used, the observations made there and the works written, see. Sayılı, *The Observatory in Islam*, 289-305; Remzi Demir, "İstanbul Rasathanesinde Yapılmış Olan Gözlemler", *Belleten* 57 (1993): 161-72. On the model of the globe in the Istanbul Observatory and related to it, Taqī al-Dīn and cartography activities in the 10/16th century Ottoman and Europe, see: Aydın Sayılı, "Üçüncü Murad'ın İstanbul Rasathanesindeki Mücessem Yer Küresi ve Avrupa ile Kültürel Temaslar", *Belleten* 25 (1961): 397-445.

19 For more detailed information about Taqī al-Dīn's application of decimal system instead of sexagesimal system to astronomy and trigonometry and the trigonometric tables he created with both systems, see: Remzi Demir, *Takiyüddîn'de Matematik ve Astronomi* (Ankara: AKM Yayınları, 2000), 28-36; Remzi Demir, "Takiyüddîn İbn Ma'rûf'un Ondalık Kesirleri Trigonometri ve Astronomiye Uygulaması", *Osmanlı Bilimi Araştırmaları* 1/2 (1998): 187-209.

Despite the need for nearly 30 years of observation in the observatory to prepare an accurate *zîj*, he maintained his observations before and after the short-lived observatory. Even though he was unable to complete his observations, he was able to create two different *zîjs*.

In summary, Taqī al-Dīn al-Rāṣid was educated in both Egyptian and Damascus madrasas and had been able to get his hand on the works of Ali Kuşçu, Qādī-zāda al-Rūmī and Jamshīd Kāshī through ʿAbd al-Karīm Mehmet, the great-grandson of Ali Kuşçu. In this way, the Samarkand School of Mathematics-Astronomy and the Egyptian-Damascus mathematical traditions were harmonized through him. The synthesis that formed was manifested both in the studies he prepared at the observatory and in his other works.

Today, we have more than 20 works in total that Taqī al-Dīn al-Rāṣid wrote. He was the greatest astronomer the Ottoman Empire had trained in the fields of medicine, mechanics, optics and mathematics, especially astronomy, the scientific path to which he devoted his life. Accordingly, he has one work on medicine, two on mechanics, one on optics, sixteen on astronomy and six on mathematics.[20]

3. Taqī al-Dīn al-Rāṣid's treatise Nisab al-mutashākila fī ʻilm al-jabr wa-al-muqābala

This work, whose title can be translated as *Homologous Proportions in the Science of Algebra and al-Muqābala* consists of an introduction, three chapters and a conclusion.

In the introduction, the terms *ʻilm al-jabr* [the science of algebra] are defined one at a time after classifying the primary and the secondary terms. In addition to this, the intro discusses the concept of exponents and the growth-reduction of any known or unknown exponential quantity in terms of the values in both integer and fractional forms.

The first chapter is titled "On Calculations" and explains how to perform four operations using algebraic expressions through rules and examples. In this chapter, Taqī al-Dīn explains that the terms and expressions in which the four operations will be performed are: of the same type (e.g., x, $3x$, $10x$); of different types (x, x^2, x^3);

20 For detailed information about Taqī al-Dīn's works, see. Fazlıoğlu, "Taqī al-Dīn", 1122-3; Topdemir, "Takıyyüddin er-Râsıd", 454-5; Şeşen vd., *OALT*, I, 202-17; Şeşen vd., *OMLT*, I, 83-7.

are positive or negative; and monomial and polynomial cases. He presents these one by one and gives appropriate examples.

The second chapter is titled "About the Rules." Put more clearly, this chapter explains how the equations being discussed are ready to be solved using various methods. These methods are *al-jabr* [completion], *ḥaṭṭ* [reduction], and *muqābala* [combining like terms].

The third and last chapter of the treatise is "Algebraic Equations." and concerns six types of equations called *mufradāt* [non-complex] and *muqtarināt* [mixed combinations], which had not lost their importance or usefulness in the period between Khwārizmī and the author. Taqī al-Dīn illustrates three non-complex and three mixed combination equations using examples, following these up with two examples of equations that do not fit the six equation types and need to be converted by completion/reduction operations. Lastly, he presents cyclic (*al-dawr*) problems and explains how to convert the problem into an algebraic equation and the method for solving it.

The conclusion is presented under the title "Solutions to Common Algebraic Equations Revealing the Secrets of This Science"; this section examines four algebra problems and explains their solutions.

3.1. The Method Followed in the English Translation

(i) Because algebraic terms such as *jazr*, *ḍilʿ*, *shayʾ*, *māl*, and *kaʿb* will be expressed using notations in the mathematical evaluation section, they have not been translated but preserved as is. However, where these terms are first mentioned in the text, the closest possible equivalent will be given in square brackets. This method has been followed due to all these terms having neither a one-word nor one-to-one English equivalent. The issue of modern mathematics being mostly composed of symbols despite the work being written in the language of verbal mathematics should be added to this situation.

(ii) In places where concepts and terms are first mentioned, their original Arabic is also given in parentheses; later occurrences will only use the English equivalent. A kind of in-text dictionary has been created in this way to familiarize the reader with classical mathematical concepts. In addition, because the text uses concepts such as *manzil*, *mufrad* and *murakkab* in more than one sense, the reader may encounter different English translations.

(iii) As pronouns are used frequently in Arabic due to the language structure, and this structure is not suitable for English, the antecedents indicated by pronouns have been reflected in the translation.

(iv) In order to ensure fluency, words or expressions in parentheses have been added to some sentences.

(v) Mathematical expressions and numbers occur in quotation marks to avoid confusion with the text.

3.2. Copy Information and Method Followed for the Editio Princeps

Sources referred to as *OMLT* and *MAOS* stated one of the copies of *Nisab al-mutashākila fī ʻilm al-jabr wa-l-muqābala* to be at Oxford and two others to be at the Cairo National Library.[21] The attempt was made to access records on the copies in Cairo using both electronic and printed sources, but no information could be obtained.[22] As for the Oxford copy, all works mentioning Taqī al-Dīn's algebra book provide the copy information as "Oxford I.881/3" but no such collection name or number is found in the electronic records of Oxford University's Bodleian Library. According to the research, some manuscript collection records in this library had been renewed, and the name and number of the collection where Taqī al-Dīn's algebra work was found is now "MS. Greaves 3/3."[23] *Nisab al-mutashākila* is the third treatise of a *majmua* [compilation] of five treatises, located between Folios 39a-42b. The entire contents of the journal can be given as follows:[24]

(i) Taqī al-Dīn al-Rāṣid, *Rayḥānat al-rūḥ fī rasm al-sāʻāt ʻalā mustawī al-suṭūḥ*, 2r-29r (On sundials).

21 Cairo, Dar al-Kutub, Mīqāt 557, ff. 44v-48r; Cairo, Taymūr Riyāda 140/10, ff. 52-61; Oxford, I, 881. For more information see: Şeşen et. al., *OMLT*, I, 85-6; Boris Rosenfeld and Ekmeleddin İhsanoğlu, *Mathematicians, Astronomers and Other Scholars of Islamic Civilization and Their Works (MAOS)* (İstanbul: IRCICA, 2003), 333.

22 To search for a catalog on the website of the national library Dar al-Kutub wa al-Wathāiq al-Qawmiyya in Cairo: http://41.33.22.69/uhtbin/cgisirsi.exe/?ps=0Szb2mqJQV/ELDAR/X/60/484/X. For the index of the Taymūr Collection, see: Anonymous, *Fihrist al-Khizāna al-Taymūriyya* (Cairo: Dar al-Kutub al-Masriyya, 1948), I-IV. For scientific works in all collections of the library, see: David A. King, *Fihrist al-makhtūtāt al-ilmiyya al-mahfūzā bi-Dar al-Kutub al-Masriyya* (Cairo: Dar al-Kutub al-Masriyya, 1981-1986), I-III.

23 https://www.fihrist.org.uk/catalog/work_6686. To find out where the name of the Greaves collection comes from and to look at the copies that have been transferred to the electronic environment. https://digital.bodleian.ox.ac.uk/collections/greaves/

24 I am grateful to Taha Yasin Arslan and Mehmet Arıkan for helping me reach this majmua.

(ii) Taqī al-Dīn al-Rāṣid, *Kitāb al-Thimār al-yāniʿa fī quṭūf al-ālat al-jāmiʿa*, 30r-37r (*taʿlīq* on the use of the spherical astrolabe).

(iii) Taqī al-Dīn al-Rāṣid, *Nisab al-mutashākila fī ʿilm al-jabr wa-l-muqābala*, 39r-42v.

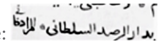
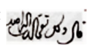
(iv) Jamshīd Kāshī, *al-Risāla al-kamālīya fī maqādīr al-ajrām al-falakīya*, 43r-54r (On calculating the distances and volumes of the planets).

(v) Anonymous, *A work on three or four geometrical problems*, in reply to Muḥammad Efendī Baqqāl-zāda 54v-59r (On planar shapes)

The introduction of the anonymously labeled last treatise informs that the work was prepared in response to Baqqāl-zāda Muḥammad Efendī's[25] (d. 1596) question. However, the anonymous author in the sentences that follow in the introduction mentions having been asked a question while he was in a group at *Dār al-Raṣad al-Ṣulṭānī* (The Istanbul Observatory). In addition to this, the phrase "Taqī al-Dīn al-Rāṣid said this" is written at the end of the work. In this case, although the treatise needs further examination, it should be considered as belonging to Taqī al-Dīn.[26]

The justification for the information given above about the journal in which the algebra treatise is located is to show that journals in the classical Islamic scientific tradition were not composed of works randomly brought together; they mostly consist of subjects and works that require and support each other and are desired to be learned together. Excluding Jamshīd Kāshī's treatise, the fact that the journal consists of Taqī al-Dīn's studies on mathematical astronomy, astronomical instruments and mathematical works is very important for understanding the methods and tools the author used in his works, especially in astronomy and mechanics. However, researching and revealing the extent and way the information given in the algebra work was used in other works is beyond the scope of this article and it requires interdisciplinary studies.

25 It is reported that he worked as a mudarris and chief doctor in the Süleymaniye Medical Madrasa but was mostly known as the chief astrologer. It is probable that he was appointed as the chief astrologer after the death of Taqī al-Dīn. It is necessary to add to this the possibility that he worked in the observatory with Rāṣid. There is information that he is proficient and famous in mathematical sciences, especially medicine and astrology. For more information see: Tuncay Zorlu, "Süleymaniye Tıp Medresesi" (Yüksek lisans tezi, İstanbul Üniversitesi, 1998), 114.

26 The part mentioning the observatory in 54v at the introduction of the treatise: 
The part where Rāṣid's name is mentioned in 57v at the end of the treatise: 

As for the method followed for the *editio princeps*, a few things need to be pointed out:

(i) Only the accessible copy of the work (i.e., the MS. Greaves 3/3 copy) has been used and the copy is marked with the letter "خ" in the footnotes.

(ii) As no indication exists that the copy is the author's or had been copied or compared from the author's, the errors and deficiencies in the copy are given in the footnotes and only proper expressions are reflected in the text.

(iii) In the text, both the chapter titles and important concepts are written in bold font, making it easier for the reader to follow.

(iv) In accordance with the publication of mathematical works, each mathematical definition, operation, rule and example is separated paragraphically, thus avoiding a possible confusion. In addition, punctuation marks have been used in accordance with this situation.

4. Mathematical Analysis[27] and Historical Evaluation: Homologous Proportions in the Science of Algebra and al-Muqābala[28]

4.1. Introduction: Explanation of Algebraic Terms[29]

27 I am grateful to Dr. Zehra Bilgin for reading and evaluating this section.

28 It should be emphasized that Taqī al-Dīn Rāṣid named the algebra book "*Nisab al-mutashākila*", that is, "homogeneous/same type ratios". Because, if the name of the work and its underlying meanings are understood, what it tries to highlight and what it emphasizes throughout the work can be realized in a better way. The concept of "ratio" is one of the most frequently used concepts throughout the text. This concept is applied at every stage, from the relationship between terms expressing the unknown, to multiplication and division, from completion (algebra) to reduction (*ḥaṭṭ*). For example, the author expresses the relationship between the thing/root (*shay'/jazr*), square (*māl*) and cube (*ka'b*) as follows: "The square becomes internal term (like internals in proportion) in the ratio between cube and root and this ratio is the secret of extracting the unknowns in the science of algebra." As can be seen, knowing the proportion between the types according to the author is the key to solving equations, so to speak. Although it is not a case before that Taqī al-Dīn Rāṣid put forward the concept of ratio in the science of algebra and gave such a name to the work of algebra, it is considered normal when the entire history of classical mathematics is taken into account. Because, when the number theory (*'ilm al-'adad*), calculation (*'ilm al-ḥisāb*), theoretical geometry (*al-handasa*) and applied geometry (*al-masāḥa*) books in the tradition are examined, it is understood how important the aforementioned concept occupies. Ratio becomes even more important, especially in the tradition of oral (*hawā'ī*) calculation; together with multiplication and division, it forms the legs of the field. For detailed explanations about this situation in the context of the author and the work, see: İhsan Fazlıoğlu, "Hesap", *DIA*, XVII, 257-60.

29 Almost all separate algebra works and algebra sections of general mathematics books, including *al-Kitāb al-Mukhtaṣar fī Ḥisāb al-Jabr wal-Muqābala*, the first separate algebra book of al-Khwārazmī known in history, begin with the introduction of basic algebraic terms. It is certain that the content of this entry changes according to the type, volume, addressee and period of the work. As in this work,

4.1.1. Primary Terms[30]

jazr or dil':[31] $a \in \mathbb{R} \setminus \{0\}$ and $a.a = a^2 \Rightarrow$ Each a is a jazr or dil'

other works in the tradition usually do this part under the title of introduction. In the mathematical tradition of Islamic civilization, in order to examine the relevant part in the algebra book or chapters that lead the course of the science of algebra, respectively, al-Khwārazmī, Abū Kāmil Shujāʿ, al-Karajī, Omar Khayyam, Samawʾal al-Maghribī, Sharaf al-Dīn al-Ṭūsī, Ismail ibn Fallūs (Mardīnī), Ibn al-Bannā, Ibn al-Hāʾim, Jamshīd Kāshī and Ali Qushji can be followed: Rushdī Rāshid, *Riyādhıyyāt al-Khwarizmī: Ta'sīs Ilm al-Jabr*, trans. Nikola Faris (Beirut: Markaz Dirāsāt al-Wahda al-Arabiyya, 2010), 168; Roshdi Rashed, Abū Kāmil Algèbre et Analyze Diophantienne: Édition, Traduction et Commentaire (Berlin: De Gruyter, 2012), 247; al- Karajī, al-Fakhrī fī al-jabr wa-al-muqābala, Sulaimaniya Library, Laleli 2740, ff. 30v-31r; Rushdī Rāshid and Ahmad Jabbār, Rasāil al-Khayyām al-Jabriyya (Aleppo: Ma'had al-Turāth al-Ilmī al-Arabī, 1981), 4; Salāh Ahmad and Rushdī Rāshid, al-Bāhir fī al-jabr li-Samaw'al al-Maghribī (Damascus: Matbaat al-Jāmia, 1972), 17-20; Rushdī Rāshid, al-Jabr wa-al-handasa fī qarn al-thānī ashar: Muallafāt Sharaf al-Dīn al-Tūsī, trans. Nikola Faris (Beirut: Markaz Dirāsāt al-Wahda al-Arabiyya, 1998), 448-9; İsmail Mardīnī, Nisāb al-habr fī ḥisāb al-jabr, Landberg 199, ff. 3r-3v; Ibn al-Bannā al-Marrākuşī, "Kitāb al-Jabr wa-al- muqābala", Ta'rīkh ilm al-jabr fī al-a'lam al-Arabī, ed. Ahmad Salim Saidān (Kuwait, 1986), II, 506-507; Ibn al-Hāʾim, al-Mumti' fī Sharh al-Muqni', Chester Beatty 3881, ff. 2v-7r; Jamshīd Kāshī, Miftāh al-Hussāb, thq. Nādir Nablūsī (Damascus: Matbaat al-Jāmia, 1988), 392-394; Ali Kuşçu, Risāla al-Muḥammadiya fī al-ḥisāb, Laleli 2715, ff. 111r-111v.

30 The earliest dated work in which these terms, which are indispensable for the science of algebra, are classified as primary and secondary degree monomials and represent a regular structure is the algebra book *al-Mumti'* by Ibn al-Hāʾim. The author's reason for making such a distinction should be sought in how algebra is verbally expressed because, starting from the 4th-degree monomial where the secondary terms begin, the concepts that define the primary terms repeat with appropriate combinations, and no new conceptualizations are used. That's why the first three terms/concepts are primary and the next are secondary. For a detailed review, see Ibn al-Hāʾim, *al-Mumti'*, ff. 2v-7r. Considering that Rāṣid grew up in the scientific environments in the triangle of Cairo, Damascus, and Istanbul, it would have been normal for him to have been influenced by the Egyptian mathematical tradition to which Ibn al-Hāʾim also belonged. In addition, the reference to Ibn al-Hāʾim at the end of the chapter on fractions strengthens the assumption that Rāṣid had read his works and been influenced by him.

31 The term *jazr*, which expresses the square root of any number, and the term *shay'*, which expresses the variable in the problem or equation were used interchangeably, especially in the first centuries of the Islamic mathematical tradition. The probable reason for this attitude is that *shay'* (e.g., *x*) is also the *jazr* [square root] of the equation. While the concept of *shay'* has been present in theology and philosophical sciences since early times, its first use as an algebraic term to express an unknown is attributed to Khwārizmī. However, Khwārizmī unexpectedly makes no mention of the term *shay'* in the introduction, instead using the term *jazr* for the unknown until the first sentence of the multiplication section after introducing the types of equations and explaining them with examples. In the first sentence of the chapter on multiplication, he says, "I will tell you how to multiply *shay*'s (unknowns) that are weighted, as if he were using a term he always does. From here on out he uses this term a lot. While Abū Kāmil showed a similar attitude, al-Karajī gave other pairs of terms that seem to be synonymous with each other, stating them to signify the same meaning but to be different in a way that only the experts of this science know. His follower, Samaw'al, explained in detail that a genus-species relationship exists between all these seemingly synonymous terms. However, the author who handled this problem most to the point was Ibn al-Hāʾim. Rāṣid's emphasis on the difference between *shay'* and *jazr* can also be traced back to Ibn al-Hāʾim. The issue of the difference between *shay'* and *jazr* is explained in detail under the footnote following the title of the English translation. For more information on the above, see Rāshed, *Riyādhiyyāt al-Khwarizmī*, 180; al-Karajī, al-Fakhrī, etc. 30b–31a; Ahmad and Rāshed, al-Bāhir, 18–19; Ibn al-Hāʾim, al-Mumti', vr. 4b–5a vr. For an examination of the issue in the context of algebra produced in the Ottoman classical period, see. Elif Baga, "Osmanlı Klasik Dönemde Cebir" (PhD thesis, Marmara University SBE, 2012), 103–105.

***jazr or shay'*:** $x\in\mathbb{R}\setminus\{0\}$ and $x.x=x^2\Rightarrow$Each x is a jazr or shay

***shay'*:** $x\in\mathbb{R}\setminus\{0\}$ $\Rightarrow x$ is called as *shay*

***māl*:** $x.x=x^2$ and for each x (jazr or shay) x^2 is called māl

***ka'b and mukaa'b*:** $x.x=x^2$; for each x (*jazr* or *shay*) and each x^2 (māl), $x.x^2=x^3\Rightarrow x^3$ is called *ka'b* or *mukaa'b*

4.1.2. Secondary Terms[32]

***māl–al-māl*:** $x\in\mathbb{R}\setminus\{0\}$ and $x.x^3=x^4$ and $x^2.x^2=x^4\Rightarrow x^4$ is called *māl–al-māl*

***māl–al-ka'b*:** $x\in\mathbb{R}\setminus\{0\}$ and $x.x^4=x^5$ and $x^2.x^3=x^5\Rightarrow x^5$ is called *māl–al-ka'b*

***ka'b–ā'l-ka'b*:** $x\in\mathbb{R}\setminus\{0\}$ and $x.x^5=x^6$ and $x^3.x^3=x^6\Rightarrow x^6$ is called *ka'b–ā'l-ka'b*

- **Proportionality o** $\frac{x^1}{x^0}=\frac{x^2}{x^1}=\frac{x^3}{x^2}=\frac{x^4}{x^3}=\frac{x^5}{x^4}=\frac{x^6}{x^5}=\cdots=\frac{x^n}{x^{n-1}}$

$x\in\mathbb{R}\setminus\{0\}$, $n\in\mathbb{N}$ and

$$x=\frac{1}{2}\Rightarrow x^2=\frac{1}{4},\ x^3=\frac{1}{8},\ x^4=\frac{1}{8}\cdot\frac{1}{2},\ x^5=\frac{1}{8}\cdot\frac{1}{4},\ x^6=\frac{1}{8}\cdot\frac{1}{8},\ldots$$

32 Just as numbers are infinity, secondary algebra terms are infinite and are made using the terms square and cube. All algebraists in the tradition agree on this point. The reason why the literal expressions, spellings, pronunciations and sequences of the terms are important and that these subjects are emphasized in the works is that the science of algebra is completely verbal until the about 14th-15th century, and after this interval, both verbal expression and numerical representation are together. On discussions such as whether notation exists in the mathematical tradition of Islamic civilization, and if so, when and how it started, see: Salih Zeki, "Notation Algebrique chez les Orientaux", Journal Asiatique 9/11 (1898): 35-52. For the translation of the article, see: Remzi Demir, "Salih Zeki Bey'in *Journal Asiatique*'de Yayınlanan 'Notation Algebrique Chez les Orientaux' Adlı Makalesi", *OTAM* 15 (2004): 333-353. In this article, Salih Zeki, mentions a treatise on algebra called *Ziyādat al-masāil al-jabriyya alā al-sitta* whose author could not be found because the first leaf was lost. He says that the author of the treatise is a Turk, but does not explain the reason for this opinion. Throughout the article, he presents the algebraic notation in this treatise, which was copied in 1430. This author, whom Salih Zeki could not identify, is Ibn al-Majdī of Turkish origin, a member of the Egyptian mathematics-astronomy school. The work is the algebraic part of the author's work called *Ḥāwī al-lubāb fī sharh Talkhīs a'māl al-ḥisāb*. For copies, see: Anonymous, *Ziyādat al-masāil al-jabriyya alā al-sitta*, Süleymaniye Library, Laleli 2734, ff. 1r-17v; Ibn al-Majdī, *Hāvī al-lubāb fī sharh Talkhīs a'māl al-ḥisāb*, Süleymaniye Library, Laleli 2741, 161 ff. To look at the issue of algebraic notation from another perspective: Jeffrey A. Oaks, "Algebraic Symbolism in Medieval Arabic Algebra", *Philosophica* 87 (2012): 27-83.

33 Proportions/ratios are one of the most basic subjects in classical mathematics. This subject, which is expressed through the concept of proportionality, is a way of understanding the relationships between numbers, the name suggests. This is one of the reasons why it has an important position in both calculus and algebra. In addition, Rāṣid's use of the expression "Homologous Proportions" in the title of the treatise should be noted, as al-Karajī in almost all algebraic studies establishes the ratio between the exponential states of *shay'* [x] from 0 to infinity. It shows their relations while at the same time prepares the basis for multiplication and division of algebraic expressions.

• Powers and how they are composed

jazr = $x^1 \Rightarrow 1^{st}$- power
māl = $x^2 \Rightarrow 2^{nd}$- power
ka'b= $x^3 \Rightarrow 3^{rd}$- power
māl–al-māl = $x^4 = x^2.x^2 \Rightarrow 4^{th}$- power
māl–al-ka'b = $x^5 = x^2.x^3 \Rightarrow 5^{th}$- power
ka'b–al-ka'b = $x^6 = x^3.x^3 \Rightarrow 6^{th}$- power
māl māl–al-ka'b = $x^7 = x^2.x^2.x^3 \Rightarrow 7^{th}$- power
māl ka'b–al-ka'b = $x^8 = x^2.x^3.x^3 \Rightarrow 8^{th}$- power -or-
māl māl māl–al-māl = $x^8 = x^2.x^2.x^2.x^2 \Rightarrow 8^{th}$- power
ka'b ka'b–al-ka'b = $x^9 = x^3.x^3.x^3 \Rightarrow 9^{th}$- power -or-
māl māl māl–al-ka'b = $x^9 = x^2.x^2.x^2.x^3 \Rightarrow 9^{th}$- power

4.2. Chapter One: Calculation[34]

34 This chapter aims to prepare the reader or student before moving on to the theory of algebraic equations by showing how to do calculations with algebraic expressions (e.g., *x*, *y*, *z*). The idea of including such a section in algebra books is based on a very fundamental idea: The problems/equations encountered are diverse and many stages such as addition, subtraction, multiplication, division and exponents are required while solving them. The way to successfully pass this stage is to know that all calculations made with real numbers can also be done with algebraic expressions. It is possible to find such a section in all algebra books written since al-Khwārazmī, albeit at different levels and contents. However, as the centuries pass, the development, expansion and change in the relevant section do not go unnoticed. While al-Khwārazmī deals with multiplication, addition-subtraction and division with algebraic expressions very superficially, Abū Kāmil additionally details the operations made with square root algebraic expressions. al-Karajī is a turning point in this regard. Because the concept of "arithmetization of algebra" emerged on the algebraic attitude of al-Karajī. Although he did not break the tradition and started the calculation part with multiplication, he also added the odds-proportion, rooting of highly rooted numbers, binomial expansion and proof, series and properties of numbers. In this case, it can be said that he expanded and deepened the science of algebra to include algebraic versions of all calculus operations. His follower, Samawʾal al-Maghribī, has preserved al-Karajī 's titles, but also included the trench proofs of the same algebraic calculations. Thus, it can be thought that al-Maghribī strengthened the algebra building that his predecessor al-Karajī had expanded and raised. As for the Maghrib tradition, it is seen that Ibn al-Bannā exhibited a similar attitude to al-Karajī in his work on algebra, but focused on the applications of algebraic calculation rules through examples. In addition, Ibn al-Bannā gives the basic terms and six equation types in the algebra and reaction part of his summary work *Talkhīs a'māl al-ḥisāb*, which includes both the known and the unknown calculations, then gives addition and subtraction operations, unlike the previous ones, then multiplication and goes to the partition. Combining Maghrib and Mashriq mathematics within the Egyptian mathematical tradition, Ibn al-Hāʾim starts algebraic calculations with addition and subtraction and, unlike Ibn al-Bannā, gives the reason for this. According to him, "if the calculation with known numbers is started with addition and subtraction for pedagogical reasons, the same thing should be done here". Besides, instead of binomial expansion and series in al-Karajī's book, he introduces polynomial rooting and indefinite analysis. As for Rāṣid, since the work is both concise and introductory, he only places bets on addition, subtraction, multiplication and division, respectively. The point that should be noted here is that Rāṣid followed Ibn al-Hāʾim's order of operations in his work, and operations like him, algebraic

4.2.1. Addition

4.2.1.1. If there are no negative terms:

• Adding terms of the same type/power (x,$3x$,$10x$…) is similar to adding numbers.

• Adding terms of different types/powers (x, x^2, x^3…)

a, b $\in \mathbb{R}\backslash\{0\}$ and $ax+bx^2=ax+bx^2$

$3x+4x^2=3x+4x^2$

4.2.1.2. If there are negative terms:

• Adding terms of the same type (x,$3x$,$10x$…)

If only one part has a negative term:

$(5x^2\text{-}2x)+3x^2=(5x^2+3x^2)\text{-}2x=8x^2\text{-}2x$

If both parts have a negative term:

$(4-5x)+(6-3x)=(4+6)-(5x+3x)=10-8x$

• Adding terms of different types (x, x^2, x^3…)

No example is provided.

4.2.2. Subtraction

• Subtracting terms of the same type (x, $3x$, $10x$…) is the same as with numbers.

• Subtracting terms of different types (x, x^2, x^3…)

a, b $\in \mathbb{R}\backslash\{0\}$ and **$ax-bx^2=ax-bx^2$** or **$bx^2-ax=bx^2-ax$**

Example: $3x-4x^2=3x-4x^2$ or $4x^2-3x=4x^2-3x$

a, b, c $\in \mathbb{R}\backslash\{0\}$ and **$(ax^2-bx)-cx=ax^2-(bx+cx)=ax^2-[x(b+c)]$**

Example: $(7x^2-x)-5x=7x^2-(x+5x)=7x^2-6x$

a, b, c, d $\in \mathbb{R}\backslash\{0\}$ and **$(ax^3-cx)-(bx^2-d)=(ax^3+d)-(bx^2+cx)\Rightarrow$**

Example: $(5x^3\text{-}3x)\text{-}(4x^2\text{-}2)=(5x^3+2)\text{-}(4x^2+3x)$

terms and expressions of the same type – different types , being positive-negative, mononomial-polynomial, separating the main headings into subheadings. To reach the information given above and more, see: Rāshed, *Riyādhıyyāt al-Khwarizmī*, 180-90; Rashed, *Abū Kāmil Algèbre*, 281-319; al-Karajī, *al-Fakhrī*, 31v-48r; Ahmad and Rāshed, *al-Bāhir*, 22-71; Ibn al-Bannā, *Talkhīs a'māl al-ḥisāb*, thq. Mohammed Suwaysī (Tunisia, 1969), 73-7; Ibn al-Hāʾim, *al-Mumti'*, ff. 9v-32v.

4.2.3. Multiplication

• For cases where the factors are monomials:

$\mathbf{a, b,} \in \mathbb{R} \setminus \{0\}\ \mathbf{a \, . \, b} = x \Rightarrow \frac{\mathbf{a}}{x} = \frac{\mathbf{1}}{\mathbf{b}}$

$\mathbf{a, b,} \in \mathbb{R} \setminus \{0\}$ **and** $\mathbf{n} \in \mathbb{N}$ **and also** $\mathbf{a}x^{\mathbf{n}}\mathbf{.b=(a.b)\ x^{n}}$

$\mathbf{a, b,} \in \mathbb{R} \setminus \{0\}$ **and** $\mathbf{n, m} \in \mathbb{N}$ **and also** $\mathbf{a}x^{\mathbf{n}}\mathbf{.b}x^{\mathbf{m}} = \mathbf{(a \, . \, b)\ x^{(n+m)}}$

Example: $4.(2x)=8x$

• For cases where the factors are polynomials:

$\mathbf{a, b, c, d} \in \mathbb{R} \setminus \{0\}$ **and** $\mathbf{n, m} \in \mathbb{N}$ **and**

$\mathbf{(a+b}x^{\mathbf{n}}\mathbf{).(c+d}x^{\mathbf{m}}\mathbf{) = a \, . \, c + (a \, . \, d)}\, x^{\mathbf{m}} \mathbf{+ (b \, . \, c)\ x^{n} + (b \, . d)\ x^{(n+m)}}$

Example: $(5+2x).(6+3x)=30+15x+12x+6x^2=30+27x+6x^2$

• For cases with positive and/or negative factors:[35]

(+).(+)=(+)	(-).(-)=(+)
(+).(-)=(-)	(-).(+)=(-)

Example 1: $(5-2x)\times(6-3x)=30-15x-12x+6x^2=30-27x+6x^2$

Example 2: $(5+x)\times(10-x)=50-5x+10x-x^2=50+5x-x^2$

• For cases where one of the factors is a fraction:

$\mathbf{a, b,} \in \mathbb{R} \setminus \{0\}$ **and** $\mathbf{n, m} \in \mathbb{N}$ **where** $\mathbf{a}x^{\mathbf{n}}.\frac{\mathbf{1}}{\mathbf{bx^m}} = x^{\mathbf{n-m}}.\left(\frac{\mathbf{a}}{\mathbf{b}}\right)$

4.2.4. Division

4.2.4.1. Single-term division

Three cases can occur when dividing any primary and secondary term into another one. Either the term is divided by itself, or by a term whose exponent is smaller than itself, or by a term whose exponent is greater than itself. Therefore, the issue of dividing a single term into another single term is one of these three types.

35 This is one of the remarkable rules in the history of mathematics. It can be said that this rule, which concerns all fields of mathematics except for theoretical geometry (*al-handasa*), is known in other ancient civilizations, especially in the ancient Mesopotamian civilization, but it is only used in problem solutions and its proof is not presented as a rule. As for the mathematical tradition of Islamic civilization, it is expressed as a rule in the multiplication chapter of the algebra book of al-Khwārazmī, which is the earliest work that has survived, but it is not proved. In fact, the work, as al-Khwārazmī stated in the introduction, appeals to the general reader and is written in accordance with this level. Mathematicians who came later also included this rule in their calculus or algebra works.

• Division by the term itself

$$\mathbf{a, b, \in \mathbb{R} \setminus \{0\}\ a>b, n \in N\ \ where}\ \frac{\mathbf{a}x^{\mathbf{n}}}{\mathbf{a}x^{\mathbf{n}}} = \mathbf{1},\ \ \frac{\mathbf{a}x^{\mathbf{n}}}{\mathbf{b}x^{\mathbf{n}}} = \frac{\mathbf{a}}{\mathbf{b}}\ \ \mathbf{and}\ \ \frac{\mathbf{b}x^{\mathbf{n}}}{\mathbf{a}x^{\mathbf{n}}} = \frac{\mathbf{b}}{\mathbf{a}}$$

• Division of a higher-order term by a lower-order term

$$\mathbf{a, b, \in \mathbb{R} \setminus \{0\}\ n, m \in N, n>m\ and}\ \frac{\mathbf{a}x^{\mathbf{n}}}{\mathbf{b}x^{\mathbf{m}}} = x^{\mathbf{n-m}}.\left(\frac{\mathbf{a}}{\mathbf{b}}\right)$$

Examples: $\frac{6x^3}{2x^2} = x^{3-2}.\left(\frac{6}{2}\right) = 3x$ and $\frac{6x^3}{9x^2} = x^{3-2}.\left(\frac{6}{9}\right) = \frac{2}{3}x$

• Division a lower-order term by a higher-order term

$$\mathbf{a, b, \in \mathbb{R} \setminus \{0\}\ n, m \in N, n<m\ \ where}\ \frac{\mathbf{a}x^{\mathbf{n}}}{\mathbf{b}x^{\mathbf{m}}} = \frac{\mathbf{1}}{x^{\mathbf{m-n}}} \cdot \left(\frac{\mathbf{a}}{\mathbf{b}}\right)$$

Examples: $\frac{5x^2}{5x^3} = \frac{1}{x^{3-2}} \cdot \frac{5}{5} = \frac{1}{x}$, $\frac{8x}{2x^3} = \frac{1}{x^{3-1}} \cdot \frac{8}{2} = \frac{1}{x^2} \cdot 4$, $\frac{8x}{12x^5} = \frac{1}{x^{5-1}} \cdot \frac{8}{12} = \frac{1}{x^4} \cdot \frac{2}{3}$

4.2.4.2. Division of a single term or many terms into many terms

It's a trade secret, and anyone who confines themself to the six types of equations doesn't need it.

4.3. Chapter Two: The Rules

4.3.1. al-Jabr[36]

36 There is no conflict about the "*al-jabr*" word being the name of this science throughout the classical algebra tradition since al-Khwārazmī's work was written. However, there are different opinions about whether the same word also expresses some of the equation solving techniques. al-Khwārazmī uses the concept of "*al-jabr*" as in the concept of "thing" without any explanation and includes it in the last paragraph of the division with algebraic expressions. Accordingly, he expresses the operation of using the verb form of the word "*al-jabr*". From here and right after that in the section where he describes six types of equations, it is seen that he uses this concept in the sense of "positing". As a result, al-Khwārazmī uses the concept of "*al-jabr*" to express the positive state of whichever expression in the equation is negative, adding to both sides of the equation, thus making the terms of the equation positive. Abu Kamil continues the same attitude. Al-Karajī says that there is a need for a definition of the concept of "*al-jabr*" under the title of six types of equations and gives the definition that expresses the positivization of the equation terms. Al-Maghribī uses this concept very rarely and equally in the sense of making positive. While Mardinī also maintains the al-Maghribī attitude, he says that "*al-jabr*" has two meanings in the *al-Tajnīs* of Hanafi jurist Sirāj al-Dīn al-Sajāwandī, one of which is positivization, and does not mention the other meaning. The possible reason for this is that he prefers the word "al-takmīl" for the other meaning "completion". On the other hand, while Ibn al-Bannā expresses both the positivization of the equation terms and the problem-solving method with the concept of "*al-jabr*" in his algebra, in his *Talkhīs*, "*al-jabr*" is to complete the simple fraction to "one". As for Ibn al-Hāʾim, he offers a summary of the evolution of the concept of "*al-jabr*", saying that "the word "*al-jabr*" is sometimes used as the opposite of "*al-ḥaṭṭ*" [reduction] and sometimes as the opposite of "*al-muqābala*", and sometimes to express this science itself". However, throughout his work, he uses the concept of "*al-jabr*" both to make negative terms positive and to complete it (total) if the coefficient of the unknown with the

For *a, b,* $\in \mathbb{R} \setminus \{0\}$ n $\in \mathbb{N}$, *a<b*, in the equation $\left[\left(\frac{a}{b}\right) \times x^n\right]$

to make change the coefficient of (x^n) equal to 1 apply the following

$\left[\frac{1}{y} = \frac{\frac{a}{b}}{1} \Rightarrow y = \frac{\boldsymbol{b}}{\boldsymbol{a}}\right]$ and multiply all terms in the equation by $\frac{\boldsymbol{b}}{\boldsymbol{a}}$

Example: $\frac{3x^2}{4} \cdot y = x^2 \Rightarrow y = \frac{1}{\frac{3}{4}}$ becomes $y = 1 + \frac{1}{3}$

When substituting this for *y* $\left[\frac{3\mathrm{x}^2}{4} \cdot \left(1 + \frac{1}{3}\right) = \mathrm{x}^2\right]$

equality is achieved,proving *y* $= 1 + \frac{1}{3}$

Another method: *For a, b,* $\in \mathbb{R} \setminus \{0\}$ n $\in \mathbb{N}$, *a<b,*

to make the leading coefficent $\left[\left(\frac{\mathrm{a}}{\mathrm{b}}\right) \cdot x^{\mathrm{n}}\right]$

multiply it by $\left[\mathbf{1} + \frac{\mathbf{1} - \frac{\mathbf{a}}{\mathbf{b}}}{\frac{\mathbf{a}}{\mathbf{b}}}\right]$

Example: The equation $\left(\frac{3x^2}{4} \times y = x^2\right)$ becomes $\left(y = 1 + \frac{1 - \frac{3}{4}}{\frac{3}{4}}\right)$

according to the above method. When substituting this value, it becomes

$$\frac{3x^2}{4} \times \left(1 + \frac{1 - \frac{3}{4}}{\frac{3}{4}}\right) = \frac{3x^2}{4} \times \left(1 + \frac{1}{4} \times \frac{4}{3}\right) = \frac{3x^2}{4} \times \frac{4}{3} = x^2$$

and appears to provide equality, so $\mathrm{y} = \frac{4}{3}$.

highest degree is less than "one" in the equation. Taqi al-Dīn Rāṣid, on the other hand, explains how to complete it to "one" if the coefficient of the unknown is less than "one" under the heading of algebra, as seen in the operations given above, and he never mentions the positive meaning of this concept throughout the text. In this case, it can be thought that the concept of "*al-jabr*" started out with the meaning of making the terms of the equations positive, and after 3-4 centuries it was used both in terms of positivization and completion, and 3-4 centuries later, the meaning of positivization began to disappear and only evolved into the meaning of completion. For resources, see: Rāshed, *Riyādhıyyāt al-Khwarizmī*, 190-216; Rashed, *Abū Kāmil Algèbre*, 321; al-Karajī, *al-Fakhrī*, etc. 48b; Ahmad and Rāshed, *al-Bāhir*, 102; Mardinī, *Nisāb al-habr*, ff. 8v; Sirāj al-Dīn al-Sajāwandī, *al-Tajnīs fī al-jabr wa-al-muqābala*, Ayasofya 3991, ff. 13v-14r; Ibn al-Bannā, "Kitāb al-jabr wa-al-muqābala", 542-544,557; Ibn al-Bannā, *Talkhīs*, 56, 74-75; Ibn al-Hāʾim, al-Mumti', 2a, vr. 49b-50a.

4.3.2. al-Ḥaṭṭ [37]

a, b, $\in \mathbb{R} \setminus \{0\}$ n $\in \mathbb{N}$, a>b, in the equation $\left[\left(\frac{a}{b}\right) \cdot x^{n}\right]$

o make change the coefficient of (x^n) equal to 1 apply the following

$\left[\frac{y}{1} = \frac{1}{\frac{a}{b}} \Rightarrow y \cdot \left(\frac{a}{b}\right) = 1\right]$ and multiply all terms in the equation by $\left(\frac{\mathbf{b}}{\mathbf{a}}\right)$

but when $\left[(b = 1) \Rightarrow \frac{a}{b} \cdot \frac{1}{a} = 1\right]$ so multiply all terms of the equation by $\frac{\mathbf{1}}{\mathbf{a}}$

Example: $x^2 + \frac{x^2}{3} = \frac{4x^2}{3} \quad \Rightarrow \quad 4 > 3 \textit{ and } \frac{1}{\frac{4}{3}} = \frac{3}{4} \quad \Rightarrow \quad \frac{4x^2}{3} \cdot \frac{3}{4} = x^2$

Another method: For *a, b,* $\in \mathbb{R} \setminus \{0\}$ n $\in \mathbb{N}$ where a>b, to make the coefficient in the expression $\left[\left(\frac{a}{b}\right) \times x^{n}\right]$ is 1, multiply it by $\left(\mathbf{1} - \frac{\frac{\mathbf{a}}{\mathbf{b}} - \mathbf{1}}{\frac{\mathbf{a}}{\mathbf{b}}}\right)$

37 It is to reduce to 1 if the coefficient of the unknown with the largest degree in the equation is greater than 1. The dictionary meaning of the word "*ḥaṭṭ*" is similarly to lower, lower and lower. The interesting aspect of this concept is that it belongs to the Maghrib mathematical tradition as far as it can be determined. Because when the algebraic works written in the east from the beginning of the algebra tradition are examined, it is seen that the word "*radd*" is preferred for this process. The earliest dated work in which the concept of "*ḥaṭṭ*" was used is Ibn al-Yāsamīn's (d. 1205) *al-Urjūza*, a verse algebra work. Here, Ibn al-Yāsamīn states that if the coefficient of square in the equations is greater than 1, the "*ḥaṭṭ*" operation should be performed, and if it is smaller, the "*jabr*" operation should be performed. In addition, if there are negative expressions in the equation, it also refers to the concept of "*jabr*", which means making positive. Ibn al-Mun'im, Ibn al-Bannā and al-Qalasādī maintain the same attitude in their account books in the expression of the reduction process with the word "*ḥaṭṭ*" after Ibn al-Yāsamīn. Combining the traditions of eastern and western algebra and calculus, Ibn al-Haim gives all the concepts used in Islamic geography, such as "*jabr*" and "*takmīl*" for completion, "*ḥaṭṭ*" and "*radd*" for reduction, but he seems to be preferred the concepts of "*jabr*" and "*ḥaṭṭ*" which are the concepts of the Maghribī tradition. As in the previous issues, it can be thought that Rāsıd exhibited a very similar stance with Ibn al-Hāʾim, and was also influenced by the Maghrib calculus-algebra tradition in general. For resources, see: Ibn al-Yāsamīn, *Manzūmāt Ibn al-Yāsamīn fī aʿmāl al-jabr wa-al-ḥisāb*, thq. Jalāl Shawkī (Kuwait, 1988), 43; Ibn Mun'im al-Abdarī, *Fiqh al-ḥisāb*, thq. Idris Murābit (Rabat: Dār al-Emān, 2005), 341; Ibn al-Bannā, *Talkhīs*, 75; al-Qalasādī, Kashf al-asrār an ilm al-khurūf al-ghubār, thq. Muhammad Suwaysī (Tunisia, 1988), 74-75; Ibn al-Hāʾim, al-Mumti', ff. 49r-49v.

4.3.3. al-Muqābala[38]

For $a, b, c, d, \in \mathbb{R} \setminus \{0\}$ and $n, m \in \mathbb{N}$ where $\frac{a}{d}x^n + bx = c$

or $\frac{a}{d}x^n = c$ or $\frac{a}{d}x^n = bx^m \Rightarrow$

$$x^n + bx \cdot \frac{d}{a} = c \cdot \frac{d}{a} \quad \text{or} \quad x^n = c \cdot \frac{d}{a} \quad \text{or} \quad x^n = bx^m \cdot \frac{d}{a}$$

For $a < d \Rightarrow x^n + bx = c + \frac{dx^n - ax^n}{d}$ or $x^n = c + \frac{dx^n - ax^n}{d}$

or $x^n = bx^m + \frac{dx^n - ax^n}{d}$

For $a > d \Rightarrow x^n + bx = c - \frac{ax^n - dx^n}{d}$ or $x^n = c - \frac{ax^n - dx^n}{d}$

or $x^n = bx^m - \frac{ax^n - dx^n}{d}$

38 The other concept, which is in the name of the science of algebra and is also the name of one of the equation solving techniques, is "*muqābala*". In fact, the vital importance of "al-*jabr*" and "*al-muqābala*" techniques in solving equations is the possible reason for naming al-Khwārazmī's book as "Algebra and Muqābala". It should be noted here that this operation can be done when there are mutually identical terms on the right and left sides of the equation. Because this is the point that separates this concept from "al-jabr" in the sense of making the terms of equations positive. If there is a negative term on one side of an equation and another term of the same type, negative or positive, on the other side, it is possible to go to the solution without any "*al-jabr*" [positivization] operation by using the "*al-muqābala*" operation. In addition, "*al-muqābala*" technique can be used not only in addition and subtraction on both sides of the equation, but also in multiplication and division operations. After these explanations, al-Khwārazmī and Abū Kāmil used "*al-muqābala*" in the sense of bringing together the terms of the same kind on the same or different side of the equation; however, it can be said that al-Karajī means both this meaning and the situation where one or two terms are equal to one or two terms, and they are found mutually. As for Ibn al-Yāsamīn, while he continues the attitude of al-Khwārazmī and Abū Kāmil, two different meanings are seen in Ibn al-Bannā's algebra work like Karajī. Ibn al-Hāʾim, on the other hand, gives the meaning of bringing together terms of the same kind , but draws attention to the relationship between "*al-jabr*" and "*al-muqābala*" explained above, and states that these two operations intersect at one point. As for Rāṣid, he takes the concept of "*al-muqābala*" from a much broader perspective. In this case, "*al-muqābala*" (i) is evaluated together with "*al-jabr*" operation, and "*al-jabr*" and "*al-muqābala*" operations are performed together, when necessary, (ii) the operation of combining terms of the same kind in the equation is also related to "*al-ḥaṭṭ*" [reduction] and "*al-jabr*" [completion] operations, since multiplication with coefficients or division operations require doing the same operations with other terms, and the result is to combine terms of the same kind. For resources, see: Rāshed, *Riyādhıyyāt al-Khwarizmī*, 190-216; Rashed, *Abū Kāmil Algèbre*, 321; al-Karajī, *al-Fakhrī*, 48v; Ibn al-Yāsamīn, *Manzūmāt*, 43; Ibn al-Bannā, "Kitāb al-jabr wa-al-muqābala", 542-544,557; Ibn al-Hāʾim, al-Mumti', ff. 49r-49v.

Another meaning of al-muqābala:

For a, b, c, d $\in \mathbb{R} \setminus \{0\}$ and n, m $\in \mathbb{N}$ where $ax^n - bx^m = c - dx \Rightarrow$
$ax^n - bx^m + \mathbf{bx^m} + \mathbf{dx} = c - dx + \mathbf{dx} + \mathbf{bx^m} \Rightarrow \mathbf{ax^n + dx = c + bx^m}$

Example $x + \frac{x}{6} \cdot \frac{1}{4} = 87 + \frac{1}{2} \Rightarrow \frac{1}{1 + \frac{1}{6} \cdot \frac{1}{4}} = \frac{x}{87 + \frac{1}{2}} \Rightarrow \frac{24}{25} = \frac{x}{87 + \frac{1}{2}} \Rightarrow$

$$25x = 24 \cdot \left(87 + \frac{1}{2}\right) \Rightarrow x = \frac{24 \cdot \left(87 + \frac{1}{2}\right)}{25} \Rightarrow x = 84$$

4.4. Chapter Three: Algebraic Equations[39]

4.4.1. al-Mufrad [Singular/Single Term] Equations

I. For **a, b,** $\in \mathbb{R} \setminus \{0\}$ where $\mathbf{ax^2 = bx} \Rightarrow x = \frac{\mathbf{b}}{\mathbf{a}}$

Example: $3x^2 = 12x \Rightarrow x = \frac{12}{3} = 4$ then $x^2 = 16$, $3x^2 = 48$ and $12x = 48$

II. For **a, c,** $\in \mathbb{R} \setminus \{0\}$ where $\mathbf{ax^2 = c} \Rightarrow x^2 = \frac{\mathbf{c}}{\mathbf{a}}$

Example: $3x^2 = 12 \Rightarrow x^2 = \frac{12}{3} = 4$ then $3x^2 = 3.4 = 12$

39 The equations section, which can be considered the heart of algebra, is given after showing and teaching all the concepts, rules and operations that will be required in solving equations, as in most of the classical algebra books produced after Karajī. The six basic types of equations are produced by binary and triple combinations of the unknown, the square of the unknown, and the constant. Three of them, which are binary combinations, are classified as simple/*al-mufrad* equations, and the other three, which are triple combinations, are classified as combine/*al-muqtarin* equations. The earliest dated work in which the complete form of this classification has survived is the book of al-Khwārazmī. In fact, although it is possible that it was written earlier than al-Khwārazmī, a part of ʿAbd al-Hamīd ibn Turk's algebra book has survived. In Ibn Turk's work, simple equations equations, and all combine equations but he makes his proofs under the concept of "logical necessity" so that the coefficient of square's is 1. In almost all algebra books written after that, first of all, these six equation patterns are explained by referring to al-Khwārazmī. However, according to the type, volume and target audience of the work, this is enough or different equations are put forward. In fact, as in Omar Khayyām and Sharaf al-Dīn Ṭūsī, a new classification of equations with 25 is made by adding the cube (x^3) to the combination terms to form equations, and geometric (*al-handasī*) techniques are also applied in solution and proof. Or, without making any changes in the classical equation classification, high-order equation types and indeterminate equations are explained under separate headings, their solutions are shown with examples, and the reasons for the solution are explained. As for Taqi al-Dīn Rāṣid, at the end of the division topic, it is understood that the information he gave is sufficient (for this work) and his statements about that there will be no need for more, as well as the fact that the work was written as a concise book, appeals to the beginner level of the algebra book. In this case, it is considered usual to limit himself to six classical equations and to include only the most basic information throughout the treatise. For resources, see: Aydın Sayılı, *Abdülhamid İbn Türk's Paper on Logical Necessities in Impure Equations and the Algebra of Time* (Ankara: TTK, 1985), 154-61; Rāshid and Jabbār, *Rasāil al-Khayyām al-Jabriyya*, 6-65; Rāshid, *al-Jabr wa-al-handasa*, 15-127.

III. For **b, c,** $\in \mathbb{R} \setminus \{0\}$ where **b*x*=c** $\Rightarrow$ $\boldsymbol{x} = \frac{\mathbf{c}}{\mathbf{b}}$

Example: $3x = 12 \quad \Rightarrow \quad x = \frac{12}{3} = 4$ then $3x = 3.4 = 12$

4.4.2. Muqtarin [Mixed] Equations

I. For a,b,c $\in \mathbb{R} \setminus \{\mathbf{0}\}$,where $\mathbf{a=1}$ and $\mathbf{a}\boldsymbol{x}^2\mathbf{+bx=c}$

$$\Rightarrow \boldsymbol{x} = \sqrt{\mathbf{c} + \left(\frac{\mathbf{b}}{\mathbf{2}}\right)^{\mathbf{2}}} - \frac{\mathbf{b}}{\mathbf{2}}$$

Example: $x^2 + 10x = 24 \;\Rightarrow\; x = \sqrt{24 + \left(\frac{10}{2}\right)^2} - \frac{10}{2}$ *and* $x = 2,\; x^2 = 4,$ $10x = 20, \quad x^2 + 10x = 4 + 20 = 24$

II. For a,b,c $\in \mathbb{R} \setminus \{0\}$ where $a=1$ and $c > \left(\frac{\mathbf{b}}{\mathbf{2}}\right)^{\mathbf{2}}$

$\Rightarrow$ *equation* **($\mathbf{ax^2+c=bx}$) is impossible to solve,**

but $\mathbf{c} < \left(\frac{\mathbf{b}}{\mathbf{2}}\right)^{\mathbf{2}}$ **and** $\mathbf{a}\boldsymbol{x}^{\mathbf{2}}\mathbf{+c=b}\boldsymbol{x} \;\Rightarrow\; \boldsymbol{x} = \frac{\mathbf{b}}{\mathbf{2}} \pm \sqrt{\left(\frac{\mathbf{b}}{\mathbf{2}}\right)^{\mathbf{2}} - \mathbf{c}}$

Example: $x^2 + 16 = 10x \;\Rightarrow\; x = \frac{10}{2} - \sqrt{\left(\frac{10}{2}\right)^2 - 16}$; $x = 2,\; x^2 = 4,\; 10x = 20$ and $4 + 16 = 20$

III. For a,b,c $\in \mathbb{R} \setminus \{0\}$, where $a=1$ and $\mathbf{a}\boldsymbol{x}^{\mathbf{2}}\mathbf{=b}\boldsymbol{x}\mathbf{+c}$

$$\Rightarrow \mathbf{x} = \sqrt{\left(\frac{\mathbf{b}}{\mathbf{2}}\right)^{\mathbf{2}} + \mathbf{c}} + \frac{\mathbf{b}}{\mathbf{2}}$$

Example: $x^2 = 4x + 5 \;\Rightarrow\; x = \sqrt{\left(\frac{4}{2}\right)^2 + 5} + \frac{4}{2}$ and $x = 5$

Note: If the coefficient of (x^2) is not equal to 1 in impure equations, make the coefficient of (x^2) equal to 1 by performing “al-ḥaṭṭ” [reduction] or “al-jabr” [completion], then compare with the other terms by performing the necessary operations.

Example of an "al-jabr" operation[40]

$$\frac{3x^2}{4} + 10x + \frac{x}{2} = 24$$

$$\Rightarrow \left(1+\frac{1}{3}\right)\cdot\frac{3x^2}{4} + \left(10x+\frac{x}{2}\right)\cdot\left(1+\frac{1}{3}\right) = 24\cdot\left(1+\frac{1}{3}\right)$$

$$\Rightarrow \quad x^2 + 14x = 32$$

$$\Rightarrow \quad x = \sqrt{32 + \left(\frac{14}{2}\right)^2} - \frac{14}{2} \text{ and } x = 2, \quad x^2 = 4\,,$$

$$\frac{3x^2}{4} = 3 \text{ and } 10x + \frac{x}{2} = 21; \text{ also}, 3 + 21 = 24$$

Example of "al-ḥaṭṭ" operation:

$$2x^2 + \frac{x^2}{4} + 7x + \frac{x}{2} = 24 \quad \Rightarrow \quad \frac{4}{9}\cdot\left(2x^2+\frac{x^2}{4}\right) +$$

$$\frac{4}{9}\cdot\left(7x+\frac{x}{2}\right) = 24\cdot\frac{4}{9} \text{ and } x^2 + 3x + \frac{x}{3} = 10 + \frac{2}{3}$$

$$\text{and } x = \sqrt{10 + \frac{2}{3} + \left(\frac{3+\frac{1}{3}}{2}\right)^2} - \frac{3+\frac{1}{3}}{2} \text{ and}$$

$$x = 2, \quad x^2 = 4, \quad 2x^2 + \frac{x^2}{4} = 9, \quad 7x + \frac{x}{2} = 15 \quad \text{and } 9 + 15 = 24$$

Note: "al-Jabr" and "al-ḥaṭṭ" operations can also be applied to al-mufrad [single term] equations when necessary.

***al-Dawr* [Cyclic] Problems:**

Example (1): The dove, pigeon, and partridge; The value of the dove is half the value of the pigeon plus fifteen dirhams, the value of the pigeon is one-fourth of the value of the partridge plus fifteen dirhams, the value of the partridge is one-fifth of the value of the dove plus fifteen dirhams. How much is each worth?

40 Here, Rāṣid tries to ensure that this process is well established in the mind of the student by reminding him again with examples after giving the equations, if the coefficient of square's in the equation he explained in the chapter of rules is less than 1, and reduction if it is greater than 1.

$$\text{dove} = x;\ \text{pigeon} = 2.(x-15) = 2x-30;\ \text{partridge}$$
$$= 4.[(2x-30)-15] = 4.(2x-45)$$
$$= 8x-180 \ \text{ and } \ 8x-180-15 = \frac{x}{5}$$

$$\Rightarrow 8x-195 = x-\frac{4x}{5};\ \text{thus}, 7x+\frac{4x}{5} = 195;$$

$$\text{thus } x\left(7+\frac{4}{5}\right) = 195 \quad \Rightarrow \quad x = \frac{195}{7+\frac{4}{5}} \Rightarrow x = 25$$

$$\Rightarrow \text{pigeon} = 2.(25-15) = 20 \ \text{ and partridge} = 4.(20-15)$$
$$= 20;\ \text{also } 20-15 = \frac{x}{5}, \quad 5 = \frac{x}{5}, \quad x = 25$$

Example (2):

Take three mines, one diamond, one ruby, and one garnet mine. The dealers say: "Subtract one-third of the value of the diamond from 100; the value of the ruby remains. Subtract half the value of the ruby from 100, the value of the garnet remains. Subtract one quarter of the value of the garnet from 100, and you will find the value of the diamond.

$$\text{diamond} = x, \text{ruby} = 100-\frac{x}{3} \ \text{ and garnet} = 100-\frac{100-\frac{x}{3}}{2}$$

$$\Rightarrow x = 100-\frac{100-\frac{100-\frac{x}{3}}{2}}{4}, 100-\frac{100-\frac{x}{3}}{2} = 50+\frac{x}{6},$$

$$100-\frac{50+\frac{x}{6}}{4} = 87+\frac{1}{2}-\frac{x}{6}\cdot\frac{1}{4} \quad \Rightarrow \quad x = 87+\frac{1}{2}-\frac{x}{6}\cdot\frac{1}{4}$$

$$\Rightarrow \ 87+\frac{1}{2} = x+\frac{x}{6}\cdot\frac{1}{4};\ x = \text{diamond} = 84,\ \ \text{ruby} = 72,\ \ \text{garnet} = 64$$

4.5. *al-Khātema* [Conclusion]: Solving Common Algebraic Equations Revealing the Secrets of This Science[41]

Problem (1): Multiplying one-third of a number by one-fourth of that number equals one-half of that number.

(Answer 1):

$$\text{number} = x \ \Rightarrow\ \frac{x}{3}\cdot\frac{x}{4} = \frac{\frac{x^2}{6}}{2} = \frac{x^2}{12} \ \text{ where } \ \frac{x^2}{12} = \frac{x}{2}$$

$$\Rightarrow\ \frac{x^2}{12}\cdot 12 = \frac{x}{2}\cdot 12 \ \text{ and } \ x^2 = 6x, \quad x = 6 \ \Rightarrow\ \frac{6}{3}\cdot\frac{6}{4} = \frac{6}{2}$$

Problem (2): One-third of a square is multiplied by one-fourth, where (the result) is three.

$$x^2 = x \ \text{ assuming } \ \frac{x}{3}\cdot\frac{x}{4} = 3\,,\ \ \frac{\frac{x^2}{6}}{2} = 3\,,\ \ \frac{x^2}{12} = 3,\ \ 12\cdot\frac{x^2}{12} = 12.3\,,\ \ x^2 = 36 \ \text{ and } \ x = 6$$

Note: The above problem leads to the second type of "*al-mufrad*" equations and requires taking the root of the square and its equivalent in order to reach the desired result.

$$x^2 = 36 \ \Rightarrow\ \sqrt{x^2} = \sqrt{36} \quad \text{and} \quad x = 6$$

41 Both this chapter and the period problems just before it can be evaluated as algebra applications, which have become a tradition in the classical algebra tradition to be put at the end of algebra books since al-Khwārazmī. Although there are exceptions, it is possible to come across these applications in most of the classical algebra books and chapters. The following point can be noted here: al-Khwārazmī devoted most of his book as a volume to problems and solutions that can be included in the science of *al-farāiḍ* under the title of "*kitāb al-wasāyā*" (book of wills). However, his successors, presumably, both because the subject also requires knowledge of *fiqh* and because they wanted to emphasize the independence of algebra, they thought that such problems had a different place, and they put different types of problems such as work-worker, allocation and return problems in the algebra applications section. As mentioned above, in the work of al-Khwārazmī, the book of wills, which also includes the chapter of transactions and the problems of return, has a very large place. Abu Kāmil, on the other hand, does not open a separate section for inheritance calculation and presents all kinds of problems in a wide range of subjects. After the theoretical part of the algebra book, al-Karajī puts forward a total of 255 questions and answers, which include all kinds of practical problems in five layers, under the title of "*ṭabaqāt al-masāil*". Algebra books of Khayyām, Maghribī and Tūsī are purely theoretical and do not have practical parts. At the end of the algebra book, Ibn al-Bannā, on the other hand, belongs to three different categories: problems with ten (*masāil al-ashra*), work-wage problems (*masāil al-rijāl*) and property problems (*masāil al-amwāl*). explains both rational and irrational problems and their solutions. Although Ibn al-Hāʾim's work on algebra was largely theoretical, he added a chapter with 7 problems, albeit a short one, at the end. Jamshīd al-Kāshī, on the other hand, explains the problems that can be encountered in all areas of daily life through 39 examples and different solution methods. For resources, see: Rāshed, *Riyādhıyyāt al-Khwārazmī*, 217-219, 235-284; Rashed, Abū Kāmil Algebre, 335-521; al-Karajī, al-Fakhrī, ff. 58r-113r; Ibn al-Bannā, "Kitāb al-jabr wa-al-muqābala", 556-585; Ibn al-Hāʾim, *al-Mumti'*, ff. 66r-68r; Jamshīd al-Kāshī, *Miftāh al-hussāb*, 489-586.

Problem (3): Ten divided into two parts, each part multiplied by itself, and whose result when added together equals 68.

(Answer 3):

$a + b = 10$ where $a = 5 + x$ and $b = 5 - x$

$\Rightarrow (5 + x)^2 + (5 - x)^2 = 68$ and $50 + 2x^2 = 68$

$\Rightarrow 2x^2 = 18$ and $x^2 = 9, x = 3 \Rightarrow a = 8$ and $b = 2$

Problem (4): We discard one-third of the square and three dirhams and are left with 20.

Answer (4)

$$x^2 = x \text{ assuming } x - \frac{x}{3} - 3 = 20$$

$$\Rightarrow \frac{2x}{3} - 3 = 20 \text{ and } \frac{2x}{3} - 3 + 3 = 20 + 3 \text{ and } \frac{2x}{3} = 23,$$

$$\frac{2x}{3} + \frac{\frac{2x}{3}}{2} = 23 + \frac{23}{2}$$

$$\Rightarrow x = 34 + \frac{1}{2}$$

5. In Place of the Conclusion

The reason for choosing such a title for the evaluation is the need to evaluate *Nisab al-mutashākila fī ʻilm al-jabr wa-al-muqābala* alongside other algebraic studies written previously, at the same time in order to reach a complete conclusion about it after this work. However, this is not possible due to the research of Ottoman history on mathematics being insufficient. Again, whether a relationship exists between this work and the other works from Taqī al-Dīn al-Rāṣid, or if other works even exist for revealing all the aspects is the reason all his works need to be examined over time. It's also one of the reasons for not reaching a complete conclusion. Because of these aforementioned reasons, the actual data that emerged as a result of examining his work and the estimates related to it are itemized as follows:

(i) The homologous ratios used in the title of the treatise draws attention as an expression not encountered in the titles of previous algebra studies. From this, the assumption is that the author will highlight the issue of ratios in the treatise. As a matter of fact, why he preferred this title is understood through the references he made to the subject of ratios in almost every title throughout the treatise as well as through the statement that the person who understands the ratios/proportions between homogeneous terms will be familiar with the intricacies of solving equations.

(ii) According to Taqī al-Dīn's statements, the treatise was not written upon request but as a kind of reminder or note for himself and those who would receive it after him. In this case, the work should appropriately be considered as an article in which a scholar briefly notes down certain algebraic rules without going into detail so that the rules are not forgotten but remain readily at hand. Therefore, the treatise is concise at the beginning level but is also an "*al-mufīd*" [helpful] work that includes all essential issues. Another sign of being at the beginner level is the author's statement at the end of the division chapter stating this much information to be sufficient for a treatise at this level.

(iii) Although no indication exists that the work was used as a textbook, it can be considered to have been used for training astronomers/astrologers or assistants in the observatory, given that Taqī al-Dīn was the Chief Astronomer/Astrologer of the Ottoman Court (Müneccimbaşı) and manager of the Istanbul Observatory.

(iv) When examining the classification of algebraic terms as primary and secondary terms, the explanation of the difference between the terms "*al-jazr*" and "*al-shay'*", the way the subject of algebraic calculation is handled and the use of the concept pair "*al-ḥaṭṭ*" and "*al-jabr*", the similarity to the algebra of Ibn al-Hāim (d. 815/1412) draws attention. Also of critical importance is that Ibn al-Hāim is the only name cited throughout the treatise. In this case, Ibn al-Hāim's algebraic studies can be seen as one of the treatise's dominant sources.

(v) In the work, a differentiation is observed in the meanings of the terms expressing the methods used for solving algebraic equations that had not been previously found. Accordingly, Taqī al-Dīn is not making the concept of algebra in the sense of positivization but only in the sense of completing it. On the other hand, he used the concept of "*al-muqābala*" in the sense of a kind of simplification as a result of his positivization, completion and reduction processes. The author appears to have been trying to eliminate the ongoing confusion on this subject by expressing the process of equating the coefficient of the variable with the highest degree to 1 in equations by using the concepts of "*al-jabr*" and "*al-ḥaṭṭ*" and by saying "*al-muqābala*" for any simplification and arrangement made to the other terms in an equation.

(vi) In the introduction to the chapter on algebraic equations, Taqī al-Dīn wrote, "The number of algebraic equations is considered to be six, but whoever is skilled in the use of equations and knows the secret of subtracting the unknowns using ratios is able to increase the number of equations." Thus, he approaches the discussion of whether the number of equations is limited or not from a different angle, pointing out that this situation depends on the knowledge and ability of the mathematician. From this, knowledge according to him can be concluded to be unlimited, but the important thing is how much can be known.

(vii) Perhaps the most striking part of the treatise is the discussion on division using algebraic expressions in the operation of dividing a lower-exponential algebraic term into a larger-exponential algebraic term. In the solution of this operation, the author criticizes Ibn al-Bannā al-Marrākushī (d. 721/1321) indirectly by naming one of his predecessors, Ibn al-Haim. This is because Ibn al-Hāim in his work *al-Mumti' fī Sharḥ al-Muqni'* stated two methods to exist for solving these types of operations. According to the first one and as Ibn al-Bannā mentioned in his *Talkhīs*, Ibn al-Hāim said the wording of the question should be repeated as the answer; in other words, it cannot be simplified. On this point, Taqī al-Dīn accused Ibn al-Bannā of teaching the result to be impossible even though a result is possible and Ibn al-Haim for repeating this information and uttering useless sentences. Although Ibn al-Hāim explained how to take the difference between the exponents as the second method, which Taqī al-Dīn also does and to make it the exponent of the term in the denominator, he was unable to avoid Taqī al-Dīn's criticism. From mathematics in general and from algebra in particular, Taqī al-Dīn al-Rāṣid appears to have been trying to clear unnecessary sentences stemming from the verbal style of expression that do not conform to the essence of mathematics.

6. English Translation

Homogeneous Ratios in the Science of Algebra and *al-Muqābala*

May Allah our Lord, our Protector bestow His blessings upon Qādī Taqī al-Dīn ibn Ma'rūf, a scholar (*'ālim*) who acts according to his knowledge ('āmil), a great scholar (*'allāma*), self-evident (*baḥr*), wise (*ḥabr*), genius (*fahhām*), a sheikh of Islamic sheikhs, a sultan of famous scholars, faithful to Allah's grace, and honor, may our Lord forgive him and his predecessors.

In the name of Allah, the merciful…

Praise be to Allah, who is al-Wahid [The One], who has made numbers with infinite digits. Greetings and prayers for the Prophet, his family, and distinguished companions.

I have compiled this work on algebra and *al-muqābala* as a "*al-tadhkira*" [reminder] for myself and those whom Allah wills after me, and I have arranged it with an introduction, three chapters, and a conclusion. Allah is the One who succeeds in the right and good ends.

Introduction: On the Explanation of Terms

The word ***jazr*** [root] is with the letter ج (*jim*) taking *al-fatḥa* and *al-kasra* (Arabic diacritics indicating the pronunciation "e" as in hen and "i" as in pick), then the silence of the letter ذ (dhal). It is a number whose property is to be multiplied and is also called the ***ḍil'*** [root].

Shay' [thing/variable; e.g., *x*] represent the unknown numbers. In order to arrive at a result, the *shay'* is assumed to be known, and the assumption is processed in accordance with a special method. A "weak precedence" exists between the *shay'* and the *jazr*.

The terms of *shay'* and *jazr* are each *sādiq* [true: the situation where a truth in the outside world is the same as a truth in the mind] when the unknown is assumed and multiplied by itself. However, the *jazr* is also *sādiq* when the known is assumed and has been multiplied by itself, while the *shay'* is *sādiq* when the unknown is assumed and has not been multiplied by itself.[42]

al-Māl [the square/x^2] results from the *shay'* or *jazr* being multiplied by itself.

42 Taqī al-Dīn al-Rāṣid uses the concept of logic "umum khusus min wach" [weak precence] to explain the relationship between the "thing" and the "jazr" here. Accordingly, every ($\sqrt{x^2}$) is met with the concept of both jazr and shay', every ($\sqrt{a^2}$) both jazr and dil', and every (x) shay'. This relationship can be represented in a table as follows: Another concept that needs to be explained here is the word that we express as "faithful" and used with the aorist conjugation of the root "s-d-q". The concept that appears in the classical philosophy-logic tradition as "al-sidq" means the agreement of an external truth with the one in the mind, and al-Rāṣid means exactly this. The change of the starting point of the movement between the outside and the mind, that is, the reconciliation of a truth in the mind with the outside, is expressed with the concept of "al-haqq/real".

a: constant and x: unknown

	Jazr	Shay	Dil'
a	$\sqrt{a^2}$	–	$\sqrt{a^2}$
x	$\sqrt{x^2}$	$\sqrt{x^2}, x$	–

Muka'ab and ***ka'b*** [cube/x^3] are mostly synonyms and are the result of the *jazr* being multiplied by *māl*. These three types are known as the primary exponents.

Māl al-māl [square of a square/x^4] is the result of multiplying the cube with its *ḍil' al-asghar* [the smallest root/x] or where *al-māl* is multiplied by itself. Because *al-māl* is in the middle of the ratio between *al-ka'b* and *al- jazr*, this ratio becomes the secret to extracting the unknowns in this science.

Māl al-ka'b [the square of the cube/x^5] results from multiplying *māl al-māl* by *al-jazr*.

Ka'b al-ka'b [cube of the cube/x^6] is the result of multiplying *māl al-ka'b* by *al-jazr*.

The ratio of each exponent to a lower-order exponent is the same as the ratio of a higher-order exponent with similar numerical difference to that exponent. If you need a higher-degree exponent, compare accordingly. These types and later ones are called ***al-manāzil al-far'iyya*** [second-degree exponents].

I say that these degrees are all raised, so they increase by using multiplication. It is permissible to assume *al-manāzil al-far'iyya* in fractions as well, so they are reduced by multiplication. Assuming *al-jazr* is "one-half", *al-māl* becomes "one-quarter", *al-ka'b* becomes "one-eighth", *māl al-māl* becomes "one-half eighth", *māl al-ka'b* becomes "one-quarter eighth" and *ka'b al-ka'b* becomes "one-eighth of an eighth". Compare accordingly.

The exponent is the locus of the existential order of each type in the plane of natural numbers. It is "one" for the *al-jazr*, "two" for the *al-māl*, and "three" for the *al-ka'b*. Accordingly, repeating the number also repeats the exponent. Addition is done as follows: *māl al-māl* is "four", *māl al-ka'b* is "five", and *ka'b al-ka'b* is "six". Compare accordingly.

If you want to know the expression of an exponent in the above terms, know its numerical order and divide it by twos or threes to the end. Thus, it is known by bringing those terms together or repeating the same term, or by both methods. The seventh power is *māl māl ka'b*, the eighth power is *māl ka'b ka'b*. It is also permissible to repeat *māl* as in *māl māl māl māl* but *māl ka'b ka'b* is preferred. The nineth power is *ka'b ka'b ka'b*. If another version is permissible, it would be done by repeating the three *māl* and the one *ka'b* [*māl māl māl ka'b*]. Compare accordingly.

Chapter One: About Calculations

Addition

This is either addition of the same type, such as the calculation of numbers, or the various additions made with the letter و [*waw* meaning "and/addition"]. "Three" *shay'* ($3x$) is multiplied by "four" *mal* ($4x^2$) where the number of *māl*s becomes attributed to the number of *shay*'s using و ; the others are like this.

As for the ***al-istithnā*** [exception regarding negative numbers], if the types are the same or similar and the negative is on one side, add the *al-sahīh* [positives] as presented and subtract the negative term from the total.

In the sum of "five *māl* minus two *shay'* plus three *māl*" [$5x^2 - 2x + 3x^2$], the total is "eight *māl* minus two *shay'*" [$8x^2 - 2x$].

If a negative occurs on both sides, add the positives then the negatives and subtract the sum from the total. For example, "four *jazr*[43] minus five *shay'* plus six *jazr* minus three *shay'*," the total is "ten *jazr* minus eight *shay'*" [$4y - 5x + 6y - 3x = 10y - 8x$].

As for difference of type, if the negative term is of a different type, it is absolutely done with و [add letter], otherwise only the negative term is added, then the sum is subtracted from the و.

Subtraction

As for those of the same or similar types, subtraction is the same as with constant numbers.

As for types that are different, subtraction is with the *al-istithnā*.

When "three *shay'* come out of four *māl* or vice versa", the answer is "three *shay'* minus four *māl*" or "four *māl* minus three *shay'*."

Whether the negativity is on one or both sides, add the negative terms on both sides and then subtract.

When subtracting "five *shay'* from seven *māl* minus one *shay'*" the coefficients become "six" and "seven" after the additions. One negative term disappears [the one *shay'*], and after the subtraction, "seven *māl* minus six *shay'*" remains.

43 In this sentence, root is used as a constant number.

Multiplication

Multiplication means to want the ratio of one of the multipliers to the outcome of the operation to be equal to a ratio of “one” to the other multiplier. If the multipliers are singular and are numbers, the multiplication operation is like any other number. If they are not numbers, the result of the multiplication is in units of that type. If one multiplier is a number and the other a type, the result is like the first, and the unit is the unit of that type.

“Four times two *jazr*” equals “eight *jazr*” and the rest of the types are like that.

If the multipliers are of two types and are singular, the result is still the same as before; the exponent is the sum of the exponents of these two types. If both or one of the multipliers are combined (not singular), multiply the coefficient of each type in the multiplier by the coefficient of the other types in the other multiplier one by one; the answer is found by knowing the type of each result and adding up all the result.

When multiplying “five plus two *shay’*” by “six plus three *shay’*,” we multiply “five” by “six”, which is “thirty”, then we multiply “five” by “three” *shay’* and get “fifteen” *shay’*. Then we multiply “two” *shay’* by “six”, which becomes “twelve” *shay’*, then we multiply “two” *shay’* by “three” *shay’*, which becomes “six” *māl*.” We add them up, and it becomes “thirty plus twenty-seven *shay’* plus six *māl*.”

If negatives are added to this operation, it has a rule, and that rule is to express the minuend as the positive and the subtrahend as the negative. The result from multiplying a positive term by a positive term or a negative term by a negative term is positive, and the result of multiplying a positive by a negative or a negative by a positive is negative. Once you know this, the answer comes by multiplying as before and subtracting the negative from the positive.

When multiplying “five minus two *shay’*” by “six minus three *shay’*,” we multiply as in the previous example, then add and subtract the negative. It becomes “thirty plus six *māl* minus twenty-seven *shay’*.”

When “five plus *shay’*” is multiplied by “ten minus *shay’*,” “five times ten” becomes “fifty”, “five times negative *shay’*” becomes “minus five *shay’*”, “*shay’* times ten becomes plus ten *shay’*” and “*shay’* times minus *shay’* becomes minus *māl*”. Because the “five *shay’*” is negative, subtract it from the “plus ten *shay’*”, then subtract the minus *māl*; we say the result is “fifty plus five *shay’* minus *māl*”.

As for the multiplication of ***al-munḥaṭṭa*** types [fractional, like] by the *al-marfu'* type [like], it is like the multiplication of ***al-marfu'*** types. The type of the multiplication result is of the type of whichever has the greater power of the *al-munḥaṭṭa* and *al-marfu'* types. If nothing is left (the powers of the *al-munḥaṭṭa* and *al-marfu'* types are equal) it returns to the number. Know this, you will need this information as you will also perform this operation in the division.

Division

As for dividing a singular term into a singular, division has three kinds:

The first is the division into the same type. Its result is not type, because the exponent of the quotient is the difference of the exponents of those two, and there is no difference between same exponents. The result is a number if the dividend is greater, "one" if the dividend and divisor are equal, and a fraction if the dividend is less than the divisor.

The second is the division of larger types by smaller types. The exponent of the result is equal to the difference of the exponents of the dividend and the divisor, and its result is like the first.

"Six *ka'b* divided by two *māl*" is "three *jazr*" and "six *ka'b* divided by nine *māl*" is "two *jazr* -thirds" [$\frac{2x}{3}$].

The third is the division of smaller types by larger types. The exponent of the quotient is still the difference; however, the result on the denominator side is just like the first type of division.

When dividing "five *māl* by five ka'b" the result is a *munḥaṭṭ jazr* (one over *al-jazr*).

When dividing "eight *jazr* by two ka'b" the result is "four times a *munḥaṭṭ māl*" (four over the *māl*).

When dividing "eight *jazr* by twelve *māl ka'b*" the result is "two-thirds *munḥaṭṭ māl māl*" [$\frac{2}{3x^4}$] .

I would say that this example of the small type divided by the large type is not meant to *tahqiq* [confirm] what Ibn al-Hāim[44] and others have said where "The

44 Ibn al-Hāʾim al-Miṣrī (d. 1412), who can be described as a mathematician-jurist, is the leading figure of the Egyptian mathematical tradition and has influenced Ottoman mathematics scholars for

answer is that question itself" because this statement has neither conceptual nor operational benefit apart from confirming what we have said. This is because multiplication is the proof of the correctness of division.[45]

Dividing the singular or compound into the compound is a trade secret, and those who limit themselves to the famous six equations don't need it.

Chapter Two: It's About the Rules

Al-jabr [completion] means the ratio of "one" to the desired value is equal to the ratio of the fraction you want to get complete to "one", thus getting the desired value from it. Both ones in the ratio are ***wasatayn*** [internal terms of the ratio], multiply these two ones and divide by the fractional number; multiplying by the fractional number results in the fractional number [the number to be used to complete the fraction to one] returning to "one". The existence of the proof is that the ***ṭarafayn*** [external terms of the ratio] are the result of the *musaṭṭaḥ* [multiplication] where the product of the internal term "one" is equal to itself.

"One over three over four" is "one plus one over three". The proper fraction whose numerator is "one" is also completed by multiplying it by its denominator.

Another method is to take the difference between "one" and the fraction, divide this difference by the fraction, and add the result that is above "one".

In our example, we proportioned the difference between "one" and the fraction (one-quarter) to the fraction; it became "one-third". We increased it and got "one plus one-third" which is the answer.

Al-Ḥaṭṭ [reduction] is to reveal the unknown value whose ratio of "one" is equal to the ratio of "one" to the value we want to reduce to and which is greater than "one."

centuries with his works. It can be said that one of them is Taqī al-Dīn Rāṣid. See the mathematical evaluation section for the mentioned effect. For more information about Ibn Haim, see: https://islamansiklopedisi.org.tr/ibnul-haim

45 The scholars and their works cited by the author here have been identified as Ibn al-Bannā's *al-Talkhīs* and Ibn al-Hāʾim's *al-Mumti'*. While Ibn al-Bannā states that the big type cannot be divided into the small type, Ibn al-Hāʾim says that there are two methods of the professionals in this regard. In the first, he refers to Ibn al-Bannā's *al-Talkhīs* and explains the use of the question word as the answer word, and in the second, as Rāṣid also states, the difference between the exponents in the numerator and the denominator is given to the denominator. In short, what Rāṣid is trying to emphasize here, neither what Ibn al-Bannā points out nor what Ibn al-Hāʾim shows as the first method has any usefulness. He also says that what he has revealed in the division the small types into the large types confirms this futility. For resources, see Ibn al-Bannā, *al-Talkhīs*, 77; Ibn al-Hāʾim, *al-Mumti'*, 25a.

The fraction used to reduce "one plus one-third" to "one" is "three-quarters". Because that is what comes out of dividing "one" by "one plus one-third" using the ***tasmiya*** method.[46]

Another method is to *tasmiya* [divide] the difference between the reduced number and "one" by the whole reduced number and then subtract that result from "one" the obvious answer remains.

During the completion and reduction processes, *muqābala* means to multiply the value in question with each of the terms in the equation, or to add the number in which the complement is completed to "one" over the other terms, or to subtract the number from the other terms as much as the value in which the reduced is reduced to "one".

It has another meaning, which is to increase or make positive, as in subtraction when there is a fraction or negative term on one or both sides of the equation.

I say that, when you know what I have presented for you regarding the ratio, you can complete or reduce the equations in one operation using *muqābala*.

Example: "One quarter" of "*shay'* plus *shay'* over six" (*shay'* plus *shay'* over twenty-four) equals "eighty-seven plus one-half".

Using the method of proportioning numbers, we find the ratio of "*shay'* to the sum of *shay'* and the fraction" to be equal to the ratio of the "unknown number to the equivalent number" and that the ratio of "twenty-four" to "twenty-five" is equal to the ratio of the "answer to eighty-seven plus one-half". We multiply both sides and divide the result by the second, leaving an answer of "eighty-four". This example is about reduction, the reverse is known to be about completion. Compare accordingly.

Chapter Three: About Algebraic Equations

The number of these equations is considered to be six, but whoever is adept at dealing with equations and knows the secret of revealing the ratio of unknowns may increase the number of equations.

Three of these six equations are said to be ***mufrad*** [simple] **equations**. They are the equality of two types or of a type and number:

46 Dividing a large number into a small number is called "*qismat*", and dividing a small number into a large number is called "*tasmiya*".

The first involves "*amwāl* [squares] that equal *juzūr* [roots]". Divide *juzūr* into *amwāl*.

For example: "Three *māl* equals twelve *jazr*." The result from the division is "for" which is the *jazr* and the *māl* is "sixteen". "Three" times *al-māl* becomes "forty-eight" and it equals "twelve" *jazr* where *al-jazr* equals "four".

The second involves *amwāl* that equal a number. Divide the number by the *amwāl*.

What comes out in "three *māl* equals twelve" is *māl* equals "four" and "three" times *māl* is equal to "twelve". They are the same.

The third involves *juzūr* that equal a number and its method is to divide the number into the *juzūr*.

Example: "Three *jazr* equals twelve". Divide "twelve" by the number of *jazr*, and you get "four" as the root. "Three" *jazr* is equal to "twelve" and it is equal to "twelve" opposite the equality. Whenever the coefficient of the unknown is "one", there is no need for division, the equivalent is the answer.

Al-Muqtarināt are likewise three, and one of these equality ratios consists of two types.

The first involves "*amwāl* plus *juzūr* being equal to a number".

The rule for finding the answer: Adding half the *māl* of the coefficient of the *jazr* to the number and subtracting half of the *jazr* from the square root ($\sqrt{\square}$) of the sum. What remains is the answer, which is the *jazr*.

For example: "*Māl* plus ten *jazr* equals twenty-four".

The answer is "two" and "two" is the root. *Māl* is "four" and "ten" *jazr* equals "twenty". You add it to the *māl* and "twenty-four" has been completed.

The **second** involves "*amwāl* plus the number being equal to *juzūr*".

The rule is conditional: The number must be less than half the square of the coefficient of *al-jazr*, otherwise the equation is impossible. When the equation is possible, subtract the number from the square of half of the number of *al-jazr*, then subtract the square root of the result from that half, that is, from the half of the coefficient of *al-jazr*. The answer is what remains.

Example of a possible equation: "*Māl* plus sixteen equals ten *jazr*".

The answer is "two" which is *al-jazr* and *al-māl* is "four". "*al-Māl* plus sixteen equals twelve" and "twelve is equal to ten *jazr*".

The **third** involves "*al-māl* being equal to *jazr* plus the number."

The rule: Adding the square of half of the coefficient of the *al-jazr* with the number, then adding the square root of the sum to half of the coefficient of the *al-jazr*. Thus, the answer is found that it is also root.

For example: "*al-Māl* equals "four" *jazr* plus five."

The answer is "five", and "five" is *al-jazr*.

***al-Tanbīh* [Note]:** The operations of combined equations are based on having the coefficient of *al-māl* be "one". Whenever the coefficient of *al-māl* is less than "one", complete the coefficient, and whenever the coefficient is greater than "one", reduce the coefficient. Then compare by doing the operation for the reduction or completion with all the remaining terms. You will know the truth, then complete the number with what the rule gives. It becomes true.

Example of *al-jabr* [completion]: "Three-quarter *māl* plus ten *jazr* plus one-half *jazr* equals twenty-four".

We complete the three-quarter *māl* by adding "one plus one-third to one *māl*" and multiplying "one plus one third" to each of the fractions of *al-māl* and *al-jazr* and the equivalency number (the corresponding number). You will arrive at the equation "*māl* plus fourteen *jazr* equal thirty-two". After dealing with the rule of the fourth of the equations, we find *al-jazr* "two", *al-māl* "four" and "three-quarter *māl* "three". Next, "ten plus one-half times *al-jazr* equals twenty-one" and "twenty-one plus three-quarter *māl* equals twenty-four.

Example of reduction: "Two *māl* plus one quarter *māl* plus seven *jazr* plus one-half *jazr* equals twenty-four".

We reduce "two *māl* plus one-half *māl*" to "one" *māl* using "four-ninth" and compare the reduction value by multiplying this to all the terms. The equation returns to "*māl* plus three *jazr* plus one-third *jazr* equals ten plus two-thirds." After applying the operation in the *al-jabr* example above, we get *al-jazr* "two", *al-māl* "four". "Two *māl* plus one-quarter *māl*" equal "nine" and "seven *jazr* plus one-half *jazr* equal fifteen". Nine plus fifteen equals twenty-four.

Know that the completion and reduction of operations also apply to simple equations.

Example of *al-dawr* [cyclic] problems: There are doves, pigeons and partridges. The value of the dove is equal to half the value of the pigeon plus "fifteen" dirhams. The value of the pigeon is equal to a quarter of the value of the partridge plus another "fifteen" dirhams. The value of the partridge is equal to one-fifth of the dove plus "fifteen" dirhams. How much is each worth?

We assume the value of the dove to be one *shay'* and subtract "fifteen" from it. Then we double this to "two *shay'* minus thirty". This is the value of the pigeon. We subtract "fifteen" from it and are left with "two *shay'* minus forty-five". Four times this remainder is "eight *shay'* minus one hundred and eighty". This is the value of the partridge. We subtract "fifteen" from it and get "eight *shay'* minus one hundred and ninety-five equals *shay'* over five". In other words, we obtain one-fifth of the dove's value (*shay'* minus four divided by five *shay'*). After *al-muqābala*, the equation becomes "seven *shay'* plus four-fifth *shay'* equals one hundred and ninety-five". After expanding both of these using the third *al-mufrad* equation (i.e., converting them to integers), we divide the number into the *shay'* and fractional *shay'*. This is another type of *al-jabr* and *al-muqābala*. After dividing, we get "twenty-five" which is the value for the dove. We subtract "fifteen" from "twenty-five" and "ten" remains. "Two" times "twenty" is the value of the pigeon. We subtract "fifteen" from it, and "five" remains. "Four" times "five" is the value of the partridge, which is "twenty". Subtracting "fifteen" from "twenty" folios "five", which is "one-fifth" the value of the dove. Compare with these examples. Everything under the rules of these six equations needs practice in terms of operations, dexterity, and weighing of thought.

Another equation from *Dawriyyāt*: There are "three" mines, one of diamonds, one of rubies, and one of garnets. Its sellers say, "subtract one-third of the value of the diamond from "hundred" and the value of a ruby remains. Subtract half the value of the ruby from "hundred" and the value of the garnet remains. Subtract "one-forth" of the value of the garnet from "hundred" and you get the value of the diamond."

We assume the value of the diamond to be *al-shay'* and subtract "one-third" *shay'* from "hundred". We get "hundred" minus "one-third" *shay'* and subtract half of that from "hundred". "Fifty" plus "one-half" remains. We subtract "one-quarter" from "hundred", and "eighty-seven plus one-half minus one-quarter *shay'* over six" remains. Equate this to *shay'* by subtracting *al-istithnā'* [exception] using *al-muqābala* to achieve "eighty-seven plus one-half equals *shay'* plus a half *shay'* over six". Then we reduce these to "*shay'* equals eighty-four" which is the value of the diamond. *al-Dawr* is continued, and the value of the ruby is found as "seventy-two" and the value of the garnet as "sixty-four" which is the answer.

Khātima **[Conclusion]: Methods of Common Algebraic Equations/ Solutions Revealing the Secrets of This Science**

Problem (1): Multiplying "one-third" of a number by "one-quarter" of that number equals "one-half" of that number.

(Answer 1): We assume the number to be *shay'* and multiply "one-third" *shay'* by "one-quarter" *shay'*. We get "one" divided by "six" *māl* because "the parts of *shay'*" times "the parts of *shay'*" are *al-amwāl* in the denominator. This value is equal to "one-half" *shay'*, multiplying the part of the *māl* by "twelve" and completing (*al-jabr*) it to "one" *māl*, it becomes exactly "one" *māl*. We compare this product with its equivalent of "one" *māl* equals "six" *shay'*; we divide *shay'*s into *al-māl*, and *shay'* becomes "six"; "one-third" of "six" multiplied by "one-quarter" of "six" is equal to "one-half" of "six".

Problem (2): "One-third" *māl* multiplied by "one-quarter" *māl* equals "three".

(Answer 2): Assume *māl al-māl*; multiply this by "one-third" and "one-quarter" as in the previous question. "One-half" of "one-sixth" *māl* equals "three" in this question. We complete it by multiplying the equal sides by "twelve", *al-māl* is equal to "thirty-six"; we take the square root, and the answer is "six".

al-Tanbih **[Note]:** We took the square root here because it is the square from dividing the second of the simple equations and does not change with division here. The result of the division is "thirty-six" *māl*; we take the square root to be that number. This needs to be noted.

Problem (3): "Ten" is divided into "two" parts. Each part is multiplied by itself, and the results are summed, giving "sixty-eight".

(Answer 3): Assuming one of the parts is "five plus *shay'*" and the other is "five minus *shay'*", we square each and add them. It becomes "fifty plus two *māl* equals sixty-eight"; we drop the common one, and "two *māl* equals eighteen" remains. "One *māl* equals nine" and its square root is "three". We subtract the "three" from one of the two "five", and "two" remains. We add "three" to the other, making "eight". So, "two" and "eight" are two parts of "ten".

Problem (4): From the *māl* we subtract "one-third" of the "*māl* plus three dirhams", leaving "twenty".

(Answer 4): We assumed *māl* as *shay'* and subtract "one-third" from it. "Two-third" *shay'* remains, from which we subtract "three", leaving "two-third *shay'* minus

3 dirhams equals twenty". Complete the "two-third" *shay'* and "three"; add "three" to "twenty" using the *al-muqābala* operation. It becomes "twenty-three equals two divided by three *shay'*". Add "one half" to "two-third" *shay'* and make it "one". You get "thirty-four plus one-half, which is the answer.

That's enough for a good thinker. Allah is the Enabler and the Helper. The treatise has been completed with the praise and success of Allah. Thanks be to Allah alone.

7. Editio Princeps

كتاب النسب المتشاكلة في علم الجبر والمقابلة
تأليف

سيدنا ومولانا العالم العامل العلامة ﷺ البحر الحبر الفهامة
شيخ مشايخ الإسلام ملك العلماء الأعلام
المتمسك بعناية الملك المعين
مولانا القاضي تقي الدين بن
معروف أعطاه الله
الخير و الشرف
وعفى عنه وعن
من سلف
م

/ [٣٩ب] بسم الله الرحمن الرحيم

الحمد لله الأحد حمداً لا تحصيه مراتب العدد، والصلاة والسلام على سيد الأنام وعلى آله وأصحابه الكرام.

وبعد

فهذه **رسالة في علم الجبر والمقابلة** ألفتها تذكرة لنفسي ولمن شاء الله من بعدي، ورتبتها على مقدمة وثلاثة أبواب وخاتمة. والله الموفق للصواب وحسن الخاتمة.

المقدمة
في
بيان الاصطلاحات

الجذر: بفتح الجيم وكسرها ثم ذال معجمة عدد من شأنه أن يضرب في نفسه ويسمى بالضلع أيضاً. **الشيء**: جملة من العدد مجهولة تفرض معلومة ليتوصل بالتصرف في فرضها على وجه مخصوص إلى معلوم، وبينه وبين الجذر عموم وخصوص من وجه. فيصدقان في فرضهما مجهولين وضرب كل منهما في نفسه. ويصدق الجذر إذا فرض معلوماً وضرب في نفسه. ويصدق الشيء إذا فرض مجهولاً ولم يضرب في نفسه.

المال: هو حاصل مضروب كل من الجذر أو الشيء في نفسه.

المكعب والكعب: مترادفان عند الأكثر وهما ما يحصل من ضرب الجذر في المال.

وهذه الأنواع الثلاثة تعرف **بالمنازل الأصلية**.

فمال المال: هو حاصل ضرب المكعب في ضلعه الأصغر أي الجذر وهو كضرب المال في نفسه، لأن المال وسط في النسبة بين الجذر والكعب، وهو سر استخراج المجهولات في هذا العلم.

مال الكعب: هو الحاصل من ضرب مال المال في الجذر.

كعب الكعب: هو حاصل مضروب مال الكعب في الجذر، ونسبة كل نوع إلى ما تحته كنسبة ما فوقه إليه. فقس على ذلك أن احتجت إلى الزيادة.

وهذه الأنواع وما بعدها تسمى **بالمنازل الفرعية**.

أقول وهذه المنازل كلها مرفوعات أي تكثر بالضرب ويجوز فرضها في جانب الكسور فتنحط بالضرب، فإذا فرض الجذر نصفاً فالربع ماله، والثمن كعبه، ونصف الثمن مال ماله، وربع الثمن مال كعبه، وثمن الثمن كعب كعبه، وقس على ذلك.

الأس: هو محل كل نوع من منزلته الوجودية في ترتيب الأعداد الطبيعية. فللجذر واحد وللمال اثنان وللكعب[1] ثلاثة. ومما تكرر لفظه تكرر أسه بحسب ذلك وكذا فما أضيف فلمال المال أربعة، ولمال الكعب خمسة، ولكعب الكعب ستة وقس على ذلك.

وان أردت معرفة الأس من النوع فاعرف منزلته العددية وأقسمها باثنين وثلاثة بالغاً ما بلغ، وعرفها بتلك الأقسام إضافة أو تكريراً وبهما، فيقع في المرتبة السابعة مال مال كعب، وفي الثامنة مال كعب كعب ويجوز فيه مال يتكرر أربع مرات والأول أولى، وفي التاسعة كعب كعب كعب وإن جاز التغيير بتكرير المال ثلاثة وإفراد الكعب وقس على ذلك.

الباب الأول
في
الحساب

الجمع: إما جمع النوع إلى مثله كحساب الأعداد او[2] جمع المختلف بالواو. فجمع ثلاثة أشياء إلى أربعة أموال عطف عدة الأموال على عدة الأشياء بالواو وكذا غيره.

وأما ما فيه استثناء **ففي متفق النوع** أن كان في جانب واحد فاجمع الصحيح كما تقدم واستثن من الجملة.

ففي جمع خمسة أموال إلا شيئين إلى ثلاثة أموال الجملة ثمانية أموال إلا شيئين. وإن كان في الجانبين فاجمع التام ثم المستثنى واستثن الجملة من الجملة كأربعة جذور إلا خمسة أشياء وستة جذور إلا[3] / [٤٠أ] ثلاثة أشياء الجملة عشرة جذور إلا ثمانية أشياء.

وأما مختلف النوع فبالعطف مطلقا إن اختلف المستثنى وإلا فيجمع المستثنى فقط ثم استثناء الجملة من المعطوف.

الطرح: أما المتفق فكسائر الأعداد وأما مختلف النوع فطرحه بالاستثناء. ففي طرح ثلاثة أشياء من أربعة أموال أو عكسه الجواب أربعة أموال إلا ثلاثة أشياء أو ثلاثة أشياء إلا أربعة أموال. وما فيه استثناء فزد المستثنى على كل من الجانبين سواء كان فيهما أو في أحدهما ثم اطرح.

١ خ: الكعب
٢ خ: و
٣ خ: إلى

ففي طرح خمسة أشياء من سبعة أموال إلا شيء، تصير بعد الزيادة ستة وسبعة. فيزول الاستثناء ويبقى بعد الطرح سبعة أموال إلا ستة أشياء.

وفي طرح أربعة أموال إلا درهمين من خمسة أكعب إلا ثلاثة أشياء، تصير أربعة أموال وثلاثة أشياء من خمسة أكعب ودرهمين. فاطرح، يبق خمسة أكعب ودرهمان إلا أربعة أموال وثلاثة أشياء.

الضرب: طلب جملة نسبة أحد المضروبين إليها كنسبة الواحد إلى المضروب الآخر.

فان كانا مفردين وهما عددان فكسائر الأعداد ولا يكون لحاصل الضرب منزلة نوعية.

وان كان أحدهما عدداً والآخر من نوع فالحاصل كالأول والمنزلة منزلة ذلك النوع. فأربعة أعداد في جذرين ثمانية أجذار، وكذا بقية المنازل.

وان كانا جنسين وهما مفردان فالحاصل أيضاً كما سبق والمنزلة لمجموع أسهما، وأن يكونا مركبين أو أحدهما فاضرب كمية كل نوع من المضروب في كمية سائر أنواع المضروب فيه واحداً بعد واحد وأعرف لكل حاصل جنسه وأجمع كمية جملة الحواصل يكن المطلوب.

ففي ضرب خمسة أعداد وشيئين في ستة أعداد وثلاثة أشياء، ضربنا خمسة في ستة بثلاثين عدداً ثم في ثلاثة أشياء بخمسة عشر شيئاً ثم ضربنا شيئين في ستة أعداد باثني عشر شيئاً ثم في ثلاثة أشياء بستة أموال. وجمعنا ذلك فكان ثلاثين عدداً وسبعة وعشرون شيئاً وستة أموال.

وان لحق ذلك استثناء فله قاعدة وهي التعبير عن المستثنى منه بالزائد وعن المستثنى بالناقص، وان الحاصل من ضرب الزائد في الزائد والناقص في الناقص زائد، ومن ضرب أحدهما في الآخر ناقص. وإذا علم ذلك فاضرب كما سبق واسقط الناقص من الزائد تلق الجواب.

ففي ضرب خمسة إلا شيئين في ستة إلا ثلاثة أشياء ضربنا كالمثال السابق ثم جمعنا واستثنينا الناقص فكان ثلاثين وستة أموال إلا سبعة وعشرين شيئاً.

وفي ضرب خمسة وشيء في عشرة إلا شيء خمسة في عشرة بخمسين وفي شيء بخمسة أشياء ناقصة وشيء في عشرة بعشرة أشياء زائدة وفي شيء بمال ناقص فيذهب من العشرة أشياء الزائدة خمسة ناقصة مقاصصة ويستثنى المال الناقص ونقول الحاصل خمسون عدداً وخمسة أشياء إلا مالاً.

وأما ضرب الأنواع المنحطة في المرفوعة فكما مرّ في ضرب المرفوعات. وجنس حاصل الضرب فضل الاثنين في أي جانب كان الفضل له وأن لم يبق شيء رجع إلى عدد فاعرف ذلك فإنك تحتاجه في القسمة كما ستحققه.

القسمة: أما للمفرد على المفرد فعلى ثلاثة مراتب:

الأولى: قسمة النوع على مثله ولا نوع لخارجها، لأن أس خارج القسمة فضل أسهما ولا فضل في المتماثلين. فهو عدد إن كان المقسوم أكثر، وواحد إن ساواه وكسر إن نقص عنه.

الثانية: قسمة النوع الأعلى على أدنى منه ونوع الخارج يعرف بأس فضل الاثنين وخارجها كالأولى.

فستة كعاب على مالين بثلاثة جذور وعلى تسعة أموال بثلثي جذر.

/[٤٠ب] **الثالثة**: قسمة نوع أدنى على أعلى منه، فأس خارج قسمته الفضل لكن في جانب الانحطاط والخارج كالأولى أيضاً.

ففي قسمة خمسة أموال على خمسة كعاب بجذر منحط.

وفي قسمة ثمانية جذور على كعبين بأربعة أموال منحطة وعلى اثني عشر مال كعب بثلثي مال مال منحط.

أقول: وهذا هو لتحقيق لا ما قاله ابن الهائم وغيره "الجواب هو السؤال" اذ لا طائل تحت هذه العبارة تصوراً ولا عملاً ويؤيد ما قلناه الضرب اذ هو برهان صحة القسمة.

وأما قسمة المفرد أو المركب على مركب فدقيق المسلك ولا يحتاجه من يتقيد بالمسائل الست المشهورة والله أعلم.

الباب الثاني
في
القواعد

الجبر: استخراج مبلغ نسبة الواحد إليه كنسبة الكسر الذي تريد جبره إلى الواحد. فالواحد وسط في النسبة فأقسمه على الكسر تلق العدد الجبري الذي إذا ضربته في الكسر عاد واحداً لقيام البرهان على أن مسطح طرفي النسبة كمسطح الوسط في نفسه.

فجزء ثلاثة أرباع بواحد وثلث وينجبر الكسر المفرد بضربه في مخرجه أيضاً.

طريق آخر: خذ الفضل بين الكسر والواحد إنسبه من الكسر وزد مثل ذلك من الواحد عليه.

ففي مثالنا نسبنا الفضل وهو ربع إلى الكسر فكان ثلثاً، زدناه بلغ واحداً وثلثاً وهو المطلوب.

الحط: استخراج مبلغ نسبته إلى الواحد كنسبة الواحد إلى مبلغ فوق الواحد نريد حطه.

ففي حط واحد وثلث إلى الواحد ثلاثة أرباع. إذ هو خارج قسمة الواحد على واحد وثلث بطريق التسمية.

وجه آخر: سم الفضل بين العدد المحطوط والواحد من جملة المحطوط واطرحه من الواحد، يبق المطلوب وهو ظاهر.

المقابلة: هي ضرب المبلغ الجبري أو الحط في كل الأجناس المتعادلة حسب ما يقتضيه الحال أو زيادة عدة واحدة على الأجناس المتعادلة كالمبلغ الذي به يتم المجبور واحداً أو إسقاط مثل ذلك منها كالمبلغ الذي يطرحه يصير المحطوط واحداً، ولها معنى آخر وهو زيادة الكسر أو الاستثناء في كل من المتعادلين إذا كان في أحدهما أو فيهما كما في الطرح.

أقول: وإذا علمت ما قدمته لك من أمر النسبة أمكنك الجبر أو الحط مع المقابلة في المتعادلين بعمل واحد.

مثاله: شيء وربع سدس يعدل سبعة وثمانين ونصفا.

وجدنا بطريق تناسب الأعداد، نسبة الشيء إلى جملة الشيء والكسر كنسبة المجهول إلى العدد المعادل. وذلك نسبة أربعة وعشرين إلى خمسة وعشرين كنسبة الجواب إلى سبعة وثمانين ونصف. فضربنا الطرفين وقسمنا الحاصل على الثاني، خرج الجواب أربعة وثمانين. وهذا في الحط ولا يخفى عكسه في الجبر فقس عليه.

الباب الثالث[٤]
في
المسائل الجبرية

المتعارف منها ستة لكن من أتقن التصرف فيها وعلم سر إخراج المجهولات منها بنسبتها أمكنه الزيادة عليها وهذه الستة يقال لثلاثة منها:

المفردات: وهي ما تعادل فيها نوعان أو نوع وعدد.

الأولى: أموال تعدل جذور. فاقسم الجذور على الأموال.

مثاله: ثلاثة أموال تعدل اثني عشر جذراً خرج بالقسمة أربعة وهي جذر، فالمال ستة عشر / [٤١أ]

٤ خ: الثاني

وثلاثة أمثاله ثمانية وأربعون مساوية لاثني عشر، أربعة وهي الجذر.

الثانية: أموال تعدل عدداً. فاقسم العدد على المال.

ففي ثلاثة أموال تعدل اثني عشر الخارج أربعة وهي مال وثلاثة أمثاله اثني عشر تعدل مثلها.

الثالثة: أجذار تعدل عدداً. وطريقها قسمة العدد على الأجذار.

مثاله: ثلاثة أجذار تعدل اثني عشر. قسمنا العدد فحصل أربعة وهي جذر وثلاثة أمثالها اثني عشر تعدل اثني عشر[٥] نظيرها ومتى كان المقسوم عليه واحداً لم يحتج إلى قسمة والمعادل هو الجواب.

المقترنات: أيضاً ثلاث وهي ما كان أحد المتعادلين منها مركباً من نوعين.

الأولى: مال وجذور تعدل عدداً.

وقاعدة استخراج جوابها بزيادة مربع نصف الجذور على العدد وطرح نصف الجذور من جذر الجملة فالباقي هو المطلوب وهو جذر.

مثالها: مال وعشرة جذور تعدل أربعة وعشرين عدداً.

الجواب اثنان وهما جذر فالمال أربعة وعشرة جذور عشرون وتجمعها إلى المال تتم أربعة وعشرون.

الثانية: مال وعدد تعدل أجذار.

فالقاعدة مشروطة يكون العدد أقل من مربع نصف الأجذار وإلا فالمسألة مستحيلة. ففي الممكن اطرح العدد من مربع نصف الجذور ثم اطرح جذر الباقي من ذلك النصف أعني نصف الجذور يبق المطلوب.

مثال من الممكن: مال وستة عشر عدداً تعدل عشرة أجذار.

الجواب اثنان وهي جذر فالمال أربعة ومع[٦] الستة عشر يكون عشرين وهي تساوي عشرة من تلك الأجذار.

الثالثة: مال يعدل جذوره وعدداً.

فالقاعدة جمع مربع نصف الأجذار إلى العدد ثم إضافة جذر الجملة إلى نصف تلك الأجذار يكن المطلوب وذلك جذر أيضاً.

٥ خ: + قسمنا العدد فحصل أربعة وهي جذر وثلاثة أمثالها اثني عشر تعدل

٦ خ: من

مثاله: مال يعدل أربعة أجذاره وخمسة.

الجواب خمسة وهو جذر.

تنبيه: أعمال المقترنات مبنية على توحد المال. فمتى نزل عن واحد فاجبره، ومتى زاد فحطه ثم قابل أي اعمل بالباقي ما عملته في الجبر أو الحط تقف على الصواب ثم كمل العدد بما تعطيه القاعدة تلق الصواب.

مثال الجبر: ثلاثة أرباع مال وعشرة أجذار ونصف جذر تعدل أربعة وعشرين.

جبرنا ثلاثة الأرباع إلى مال بواحد وثلث وضربناه في كل من كسر المال والجذور والعدد المعادل وهو معنى المقابلة فبلغت الصورة إلى مال وأربعة عشر جذراً تعدل اثنين وثلاثين وبعد العمل بقاعدة رابعة المسائل وجدنا الجذر اثنين فالمال أربعة، ثلاثة أرباعها ثلاثة، وعشرة أمثال ذلك الجذر ونصف مثله أحد وعشرون يعدل مع ثلاثة أرباع المال أربعة وعشرين.

ومثال الحط: مالان وربع وسبعة أجذار ونصف تعدل أربعة وعشرين. حططنا المالين والربع إلى مال بأربعة اتساع وقابلنا بضرب المبلغ الحطي في جملة المعلومات. فرجعت المسألة إلى مال وثلاثة أجذار وثُلث جذر يعدل عشرة وثُلثين وبعد العمل المتقدم في مثال الجبر[٧]. فالجذر اثنان والمال أربعة ومالان وربع تسعة، وسبعة أجذار ونصف خمسة عشر والجملة تعدل أربعة وعشرين.

واعلم أن العمل في الجبر والحط تجري في المفردات أيضاً.

مثال من المشكلات الدورية، يمامة، وحمامة، ودرّاجة. قيمة اليمامة نصف قيمة الحمامة وزيادة على ذلك خمسة عشر درهماً، وقيمة الحمامة ربع قيمة الدراجة وخمسة عشر أيضاً، /[١٤ب] وقيمة الدراجة خمس قيمة اليمامة وخمسة عشر أيضاً كم قيمة كل منها؟

فرضنا قيمة اليمامة شيئاً وطرحنا منه خمسة عشر ثم ضعفناه فكان شيئين إلا ثلاثين وهو قيمة الحمامة طرحنا منه خمسة عشر فبقي شيئان الا خمسة وأربعين. فأربعة أمثاله ثمانية أشياء إلا مائة وثمانين وهي قيمة الدراجة. ونطرح خمسة عشر ينتهي إلى ثمانية أشياء إلا مائة وخمسة وتسعون تعدل خمسا أعني من قيمة اليمامة وهو شيء إلا أربعة أخماس وبعد المقابلة تصير إلى سبعة أشياء وأربعة أخماس تعدل مائة وخمسة وتسعين فبثالث المفردات قسمنا العدد على الأشياء والكسر بعد بسط كل منهما أي إرجاعه إلى صحيح وهذا نوع آخر من الجبر والمقابلة فخرج بالقسمة خمسة وعشرون وهي قيمة اليمامة. طرحنا منها خمسة عشر بقي عشرة ضعفها عشرون وهي قيمة

٧ خ: الجذر

الحمامة. أسقطنا منها خمسة عشر بقي خمسة وأربعة أمثالها قيمة الدراجة وذلك عشرون. فإذا طرح منها خمسة عشر بقي خمسة وهي خمس قيمة اليمامة. فقس على هذه الأمثلة كلما كان داخلاً تحت ضوابط هذه المسائل الستة وذلك يحتاج إلى ممارسة الأعمال والحذق واجالة الفكر.

مسألة من الدوريات أيضاً: وهي ثلاثة معادن الماس وياقوت ولعل. قال بائعها «اطرح ثلث قيمة الماس من مائة تبقى قيمة الياقوت، فاطرح نصف قيمة الياقوت من المائة، تبقى قيمة اللعل. وإذا طرحت ربع قيمة اللعل من مائة تجد قيمة الماس.»

فرضنا قيمة الماس شيئاً وطرحنا ثلثه من مائة تأخر مائة إلا ثلث شيء. طرحنا نصفه من مائة فبقي خمسون وسدس. طرحنا ربعه من مائة بقي سبعة وثمانون ونصف إلا ربع سدس تعدل شيئا فبالمقابلة طرحنا الاستثناء بلغ سبعة وثمانين ونصف تعدل شيء وربع سدس. ثم حططناهما بقي شيء يعدل أربعة وثمانين وهو قيمة الماس وانقطع الدور. فقيمة الياقوت اثنان وسبعون وقيمة اللعل أربعة وستون وهو المطلوب.

الخاتمة
في
لطائف المسائل الجبرية المتداولة التي تعين على الاطلاع على أسرار هذا العلم

مسألة: عدد ضرب ثلثه في ربعه فبلغ نصف العدد.

فرضناه شيئاً وضربنا ثلثه في ربعه، بلغ نصف سدس مال. لأن أجزاء الأشياء في أجزاء الأشياء أموال منحطة، وهذا المبلغ يعدل نصف الشيء المفروض. فجبرنا جزء والمال إلى مال كامل تضربه في اثني عشر فصار مالاً كاملاً وقابلنا المعادل بذلك الضرب، فحصل مال يعدل ستة أشياء قسمنا الأشياء على المال فكان ستة ومسطح ثلثها في ربعها مساو لنصفها.

مسألة: مال يضرب ثلثه في ربعه فبلغ ثلاثة.

فاجعل المال شيئاً واضرب ثلثه في ربعه بنصف سدس مال كما مر يعدل هنا ثلاثة. فجبرنا المتعادلين بالضرب في اثني عشر فكان مالاً يعدل ستة وثلاثين أخذنا جذرها فكان المطلوب ستة.

تنبيه: إنما أخذنا هنا الجذر لأن حاصل قسمة ثاني المفردات مال وهنا لا يتغير بالقسمة فحاصل القسمة ستة وثلاثون مالا فأخذنا جذرها ليكون عدداً فليتنبه له.

مسألة: عشرة قسمت قسمين. فضرب كل قسم في نفسه وجمع فكان ثمانية وستين. فرضنا أحد القسمين خمسة وشيئاً والأخر خمسة إلا شيئاً فربعنا كلا منهما وجمعنا فكان خمسين ومالين / [٢٤أ]

تعدل ثمانية وستين فأسقطنا المشترك بقي مالان يعدلان ثمانية عشر الواحد يعدل تسعة وجذره ثلاثة نسقطها من إحدى الخمسين، يبقى اثنين ونزيدها على الأخرى، تبلغ ثمانية وهما القسمان.

مسألة: مال ألقيت منه ثلثه[٨] وثلاثة دراهم، فبقي منه عشرون.

جعلنا المال شيئاً وألقينا ثلثه بقي ثلثا شيء ألقينا من ذلك ثلاثة، بقي ثلثان إلا ثلاثة دراهم تعدل عشرين. فاجبر الثلثين بالثلاثة وزد مثل ذلك على العشرين بالمقابلة، فيكون ثلاثة وعشرين تعدل ثلثي شيء فأكملها بأن تزيد عليها نصفها تحصل أربعة وثلاثون ونصف وهو المطلوب.

۞ وفي هذه القدر كفاية للمستبصرين۞ والله الموفق والمعين۞

تمت الرسالة بحمد الله وحسن

۞ توفيقه۞ والحمد لله ۞

۞ وحده ۞

۞

٨ خ: ثلاثه

Bibliography

Primary Sources

Manuscripts

Ibn al-Hāim, *al-Mumtiʿ* fī Sharḥ al-Muqniʿ, Chester Beatty 3881 (author's copy).

Ibn al-Majdī, *Ḥāwī al-Lubāb fī Sharḥ Talkhīṣ aʿmāl al-ḥisāb*, Süleymaniye Kütüphanesi, Laleli 2741.

_______, *Ziyādat al-masāʾil al-jabriyya ʿalā al-sitta*, Süleymaniye Kütüphanesi, Laleli 2734/1.

Al-Karajī, *al-Fakhrī fī al-jabr wa-l-muqābala*, Süleymaniye Kütüphanesi, Laleli 2740.

Kuşçu, Ali, *Risāla al-Muḥammadiyya fī al-ḥisāb*, Süleymaniye Kütüphanesi, Ayasofya 2733.

Al-Mardīnī, İsmāʿīl, *Niṣāb al-ḥabr fī al-ḥisāb al-jabr*, Landberg 199.

Rāṣid, Taqī al-Dīn, *Sidrat al-muntahā*, Kandilli Rasathanesi 208.

_______, *Kitāb al-nisab al-mutashākila fī ʿilm al-jabr wa al-muqābala*. MS. Greaves 3/3, 39a-42b.

Sajāwandī, Sirāj al-Dīn, *al-Tajnīs fī al-jabr wa al-muqābala*, Süleymaniye Kütüphanesi, Ayasofya 3991.

Printed Texts

Khayyām, Omar. *Rasāil al-Khayyām al-Jabriyyat*, thk. Rushdī Rāshid wa Aḥmad Jabbār, Ḥalab, 1981.

Ibn al-Munʿim, *Fiqh al-ḥisāb*, thk. Idrīs Murābiṭ, Rabat, 2005.

Ibn al-Bannāʾ al-Marrākushī, *Talkhīṣ aʿmāl al-ḥisāb*, thk. Muhammed Süveysî, Tunus, 1969.

_______, *Kitāb al-jabr wa-l-muqābala*, Tārīkh ʿilm al-jabr fī al-ʿālam al-ʿarabī, Saʿīdān, Aḥmad Salīm, II, 500-585. Kuwayt, 1986.

Ibn Yāsamīn. *Manẓūmāt Ibn Yāsamīn fī aʿmāl al-jabr wa-al-ḥisāb*. Edited by Jalāl Shawqī. Kuwait: Muʾassasat al-Kuwayt li-l-taqaddum al-ʿilmī, 1988.

al-Qalaṣādī, *Kashf al-Asrār ʿan ʿIlm Ḥurūf al-Ghubār*, tḥq. Muḥammad Suwaysī, Tunus, 1988.

al-Kāshī, Jamshīd, *Miftāḥ al-Ḥussāb*, tḥq. Nādir Nablūsī, Dimashq, 1977.

al-Samawʾal al-Maghribī. *al-Bāhir fī al-jabr*. Edited by Ṣalāḥ Aḥmad and Rushdī Rāshid. Damascus: Wizārat al-taʿlīm al-ʿālī, 1972.

Rashed, Roshdi. Abu Kamil, *Algèbre et Analyse Diophantienne: Édition, Traduction et Commentaire*, Berlin: De Gruyter, 2012.

Rāshid, Rushdī, ed. *al-Jabr wa-al-handasa fī al-qarn al-thānī ʿashar: muʿallafāt Sharaf al-Dīn al-Ṭūsī*. Beirut: Markaz dirāsāt al-waḥdat al-ʿarabiyya, 1998..

_______, *Riyāḍiyyāt al-Khwārizmī: Taʾsīs ʿilm al-jabr*. Trans. Niqūlā Fāris. Beirut: Markaz dirāsāt al-waḥdat al-ʿarabiyya, 2010.

Sayılı, Aydın, *Abdülhamid İbn Türk'ün Katışık Denklemlerde Mantıki Zaruretler Adlı Yazısı ve Zamanın Cebri*, Ankara: TTK Yayınları, 1985.

Shaʿrānī, Munā Sanjaqdār, *Dirāsāt taḥlīliyya li-maḥṭūṭ al-ṭuruq al-saniyya fī al-ālāt al-rūḥāniyya*, Kuwait, 2002.

Secondary Sources

Books

Anonymous, Fihrist al-Khizānat al-Taymūriyyat, I-IV, Qāhirat: Dār al-Kutub al-Miṣriyya, 1948.

Demir, Remzi, Takiyüddîn'de Matematik ve Astronomi, Ankara: AKM Yayınları, 2000.

Al-Ḥasan, Aḥmad Yūsuf, Taqī al-Dīn wa al-handasat al-mīkānīkīyyat al-Arabiyyat. Ḥalab, 1987.

King, David A. Fihrist al-makhṭūṭāt al-ʿilmiyyat al-maḥfūẓat bi-dār al-Kutub al-Miṣriyya, I-III, Qāhirat: Dār al-Kutub al-Miṣriyya, 1981-1986.

Rosenfeld, Boris A. ve Ekmeleddin İhsanoğlu, Mathematicians Astronomers and Other Scholars of Islamic Civilizations and Their Works, Istanbul: IRCICA, 2003.

Sayılı, Aydın, The Observatory in Islam, Ankara: TTK Yayınları, 1988.

Şeşen, Ramazan vd., Osmanlı Matematik Literatürü Tarihi, İstanbul: IRCICA, 1999.

______, Osmanlı Astronomi Literatürü Tarihi, İstanbul: IRCICA, 1997.

Dissertations

Baga, Elif, "Osmanlı Klasik Dönemde Cebir", Doktora tezi, Marmara Üniversitesi, 2012.

Zorlu, Tuncay, "Süleymaniye Tıp Medresesi", Yüksek lisans tezi, İstanbul Üniversitesi, 1998.

Articles, Papers and Encyclopedia Articles

Aydüz, Salim, "Osmanlı Devleti'nde Müneccimbaşılık". Osmanlı Bilimi Araştırmaları 1 (1995), 159-208.

Demir, Remzi, "İstanbul Rasathanesinde Yapılmış Olan Gözlemler", Belleten 57 (1993), 161-72.

______, "Takiyüddîn İbn Ma'rûf'un Ondalık Kesirleri Trigonometri ve Astronomiye Uygulaması", Osmanlı Bilimi Araştırmaları 1/2 (1998): 187-209.

______, "Sâlih Zeki Bey'in Journal Asiatique'de Yayımlanan 'Notation Algébrique Chez les Orientaux' Adlı Makalesi", OTAM 15 (2004), 333-53.

Dosay, Melek, "Takîyüddîn'in Cebir Risalesi", Belleten 61 (1997), 301-20.

Fazlıoğlu, İhsan, "Mustafa İbn Ali el-Muvakkit", DİA, XXXI, 287-8.

______, "Taqī al-Dīn Abū Bakr Muḥammad ibn Zayn al-Dīn Maʿrūf al-Dimashqī al-Ḥanafī", The Biographical Encyclopedia of Astronomers, 1122-3, ed. Thomas Hockey vd., Newyork: Springer, 2007.

______, "Ibn Abī al-Fatḥ al-Ṣūfī: Shams al-Dīn Abū ʿAbd Allāh Muḥammad ibn Abī al-Fatḥ al-Ṣūfī". The Biographical Encyclopedia of Astronomers, ed. Thomas Hockey vd., 547, Newyork: Springer, 2007.

______, "Ali al-Muwaqqit: Muslih al-Dīn Mustafa ibn Ali al-Qustantini al-Rumi al-Hanafi al-Muwaqqit". The Biographical Encyclopedia of Astronomers, ed. Thomas Hockey vd., 33-4, Newyork: Springer, 2007.

İpşirli, Mehmet, "Çivizâdeler", DİA, VIII, 349-50.

Mevâldî, Mustafa, "Tahkîk ve dirâse mahtût Kitâbu'n-Nisebi'l-müteşâkile fi ilmi'l-cebr ve'l-mukâbele li-Takiyyüddîn b. Ma'rûf", Ebhâsu'l-Mu'temer es-senevî li-târîhi'l-ulûm inde'l-Arab, 445-70, Haleb: Ma'hedu't-Turâsi'l-İlmî el-Arabî, 2003.

Oaks, Jeffrey A., "Algebraic Symbolism in Medieval Arabic Algebra", Philosophica 87 (2012), 27-83.

Sayılı, Aydın, "Üçüncü Murad'ın İstanbul Rasathanesindeki Mücessem Yer Küresi ve Avrupa ile Kültürel Temaslar", Belleten 99 (1961), 397-445.

Şeşen, Ramazan, "Meşhur Osmanlı Astronomu Takıyyüddin Râsıd'ın Soyu Üzerine", Erdem 10 (1988), 165-71.

Taşköprülüzâde, Osmanlı Bilginleri, trc. Muharrem Tan, İstanbul: İz Yayıncılık, 2007.

Tekeli, Sevim, "Trigonometry in the Sixteenth Century: Copernicus and Taqî al-dîn", Erdem 2/4 (1986), 247-72.

Topdemir, Hüseyin Gazi, "Takıyyüddin Râsıd", DİA, XXXIX, 454-5.

Ünver, Süheyl, İstanbul Risaleleri, haz. İsmail Kara, II, İstanbul: İBB Yayınları, 1995.

Zeki Efendi, Salih. "Notation Algébrique Chez les Orientaux". Journal Asiatique 9/11 (1898), 35-52