A Mathematical Analysis of the Theory of Horizontal Sundials in the Ottoman Period: The Case of Risālah of Ruhāma

Atilla Bir,* Şinasi Acar,** Mustafa Kaçar***

Abstract: As the prayer times are determined according the sun’s position, the theory of sundials, how to draw and design them, hold a significant place in the history of Islamic astronomy. All Muslim scientists working on applied and theoretical sciences have books on this subject. Besides, as it was a part of **madrasa** curriculum, there are considerably high number of unexamined books from the beginning to the end of the Ottoman period. This study is about a manuscript titled **Risālah-i Vaz'ī Ruhā**: that we received from a second-hand book seller. Its anonymous author argues that he offers a simpler way to draw sundials designed for Istanbul. The author leaves the person willing to make a sundial free to determine the height of the gnomon and he shows a practical method to draw ruhāma with the help of goniometer and ruler prepared according to the determined height. Although the article presents the mathematical bases of the method, it is unable to verify clearly how the author derived the numeric values given in the book. To the end of our study, facsimile text and its simplified transcription are attached.

Keywords: Theory of Sundials, gnomonic, Muslim Sundials, horizontal sundials, time systems in Islam, prayer times, ruhāma.

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1. Introduction

The time period of the Sun virtually revolving around the world is defined as 1 full day. In ancient Egypt – without taking the seasons into account – the days and nights, independent of each other, are divided into 12 equal hours. This conception of time is labeled as seasonal or temporal hour. On the 41st latitude, where Istanbul is located, on the 22nd of June, the longest day is 16 hours and 22 minutes, and the shortest night is 7 hours and 38 minutes; and on the 22nd of December, the shortest day is 9 hours and 27 minutes, whereas the longest night is 14 hours and 33 minutes. If these time lengths are divided into 12, in the summers the longest day hour is 1 hour and 21.83 minutes, and the shortest night hour is 38.17 minutes; and in the winters the shortest day hour is 47.25 minutes, and the longest night hour is 1 hour and 12.75 minutes. On the 21st of March in the spring, and 23rd of September in the autumn, on the other hand, the time lengths of day and night are equal and each is 12 hours.\(^1\)

In ancient Egypt, and in the ancient world, time was determined during the daylight by measuring the shade-length of a stick (gnomon) fixed into the ground. For example, the obelisk in Sultan Ahmet Square in Istanbul, which was brought to the city in the 4th century from the Temple of Ammon in Karnak/Egypt, was a gnomon of a sundial. The hours in the night used to be determined by the rising times of certain stars or by their passage through the longitude. In addition, sandglasses and water clocks were also used as auxiliary methods to supplement day time calculation. In sandglasses, time is measured by the flow of sand in a bottle, and in water clocks, the flow of the water quantity is measured in specific time intervals. Additionally, people used candle clocks at night.

In Mesopotamia, days are divided into 12 hours of equal length. Herodotus (490-425 BC) narrates that the Greeks learned about sundials and dividing days into 12 from Babylon. In the Hellenistic period (330 BC -30), days began to be divided into 24 equal hours, as it is the case today, and especially as it is like seasonal hours of March 21st and September 23rd. Such kind of hours are called equal hours. Yet, taking midday or midnight as the beginning of the new calendar day (alafranga, zawâli), and taking sunset as the beginning of new day (alaturka, ezâni) are two different applications of this time measurement.

In the Lunar calendar or Hijri calendar, Lunar months begin with the sighting of the Moon as a thin crescent during sunset. The new month begins the new cal-

\(^1\) Atilla Bir, Mustafa Kaçar, Şinasi Acar, Güneş Saatleri Yapım Kilavuzu (İstanbul: Biryl Yayınları, 2010), p. 28.
endar day, and when the Sun’s top edge is tangent to the horizon during sunset, the hour is accepted to be as 12:00 or 0:00. The night that begins is considered not to be part of the previous day but the new one. As a result of this relatively defined day, the time span between two consecutive nights is not exactly equal to 24 hours. In the springs, it is a little longer, and in the autumns it is a little shorter. Hence, mechanical clocks need to be set to 12 manually every day during sunset.

The moment when the center of the Sun is just on the circle of the longitude is defined as midday. Timepieces which accept that moment as 12:00 are called noon-related alafranga or zawālī hour times. The word zawāl means exact midday time. To prevent possible confusions that may stem from setting the beginning of a new day at midday, usually in this hour system - as it is the case today - the day begins 12 hours before the midday, which means midnight.

Since fasting and five daily prayers are ordered by Islam, and since these deeds are closely related to specific time slots, Muslims paid considerable attention to determining the times of noon, mid- afternoon and sunrise. In addition to this characteristic which we do not observe in religions other than Islam, accumulation of experience about this subject led to the production of more exact and more accurate timepieces, which in the past used to be produced just to know the time of the day. Besides, we do not know use of any devices for determining time, by Arabs in the pre-Islamic era, neither do we know their presence at the time of the Prophet Muhammad and four caliphs who succeeded him.

The first works about horizontal sundials are written in the Abbasid period. Although sources they used for calculating and drawing sundials were from pre-Islamic civilizations such as Ancient Greek, Egyptian and Indian civilizations, Muslim astronomers made considerable theoretical and practical contributions to the development of sundials. First Muslim astronomers who wrote on sundials are Ibrāhīm al-Fazārī (2nd/8th century), Habash al- Ḥāsib (3rd/9th century), Muḥammad ibn Mūsā al-Khwārizmī (2nd-3rd/8-9th century), Muḥammad ibn Ṣabbāh (3rd/9th century), al-Farghānī (3rd/9th century), Ibn al- ‘Adamī (3rd/9th century) and Abū ‘Abd Allah Muḥammad ibn al-Ḥasan ibn Abī Hishām al- Shatawī (3rd/9-10th century). To our knowledge, the first book written in Islamic civilization about sundials is the al- Fazārī’s Kitāb al- mikyās li al- zawāl, which he wrote when he was in Baghdad, but unfortunately is lost. The oldest extant book on Islamic sundial is ‘Amal al- sā’a fi

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basīṭ al- ruhāma, which is written in the early 9th century in Baghdad and attributed to al-Khwārizmī. The most important work which clearly explains how different sundials known in the Islamic world are produced is Thābit ibn Qurra’s Kitāb fī ālāt al- sā’a allati tusammā ruhāmāt. The book is at the Kopru Library in Istanbul.

The manuscript entitled Risālah-i vaż’-i ruhāma (Book on Ruhame Construction) that we analyze in this article is the first known work written in the Ottoman period, which discuss Islamic sundials. Its author is unknown. On the inside of its cover the following phrase is seen: “Written for his Excellency Zeynel Beg” (Picture 2-a). Zeynel Beg (d. 1589) is the grandson of Asad al- din Kalanī who founded the Hakkari Emirate. He was appointed as the Amir of Hakkari by the Ottoman state, and supported by Süleyman I (1520-1566), Selim II (1566-1574) and Murad III (1574-1595). He was martyred in Merent during the Tabriz campaign; and after the conquest of the city, his corpse was brought to Hakkari (it was called Çölemerik at the time), and buried in the courtyard of Zeynel Beg Madrasa which was built by him. There is a very similar note “to his excellency Zeynel Beg”, which gives the

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6 Köprülu Library, nr. 984/1.
7 “Zeynel Beg was a loyal and brave one among Kurdish Begs in Hakkari [a city located in southeastern
impression of being written by the same person, in a corpus (majmu’a) at Topkapi Palace Library, in the collection of Hazine (n. 452), which includes writings on astronomy and astrology about Taqi al-din al-Râsid and Ahmed-i Dâ’î (Picture 2-b). The 17.5x11 cm sized hard-covered book that contains also this work includes three different works. The first one is an astronomy book about stars, and the third one is an astrology book dated 890/1485.

Though in the manuscript it is argued that drawing sundial is very easy and absolutely accurate, the text does not mention the kind of sundial. While it is stated that the sundial is designed for Istanbul (\( \varphi = 41^\circ \)), the text does not give information about the azimuth angle of the wall which is required for the vertical sundials. The fact that the values about both kind of \( \text{ghurūbī} \) hour systems (Babylonian and italic) that are used in the drawing, with \( \text{zawālī} \) hour system, are given symmetric to the meridian, and more especially that the equinoctial line is given horizontal indicate that this timepiece is a horizontal sundial. In addition, the fact that drawing the \( \text{asr-i awwal} \) and \( \text{asr-i thāni} \) curves on the same plane with both \( \text{ghurūbī} \) hour system supports this assumption. Since wall clocks are usually drawn directly onto the wall, the name \( \text{ruhāma} \), literally means marble, given to this timepiece should refer to a horizontal sundial.\(^8\)

\(^8\) There is a sundail drawing at the end of the text, but there is nothing on the drawing which makes reference to the text itself. We will present our opinions about that drawing at the end of the article.

part of Turkey], who lived in the time of Süleyman I.” (Sicilli-Osmani 5, Tarih Vakfı Yurt Yayınları, 1996, 1709). (translation made by the authors)
2. Mathematical Analysis

Since the quadrant of an equatorial sundial is parallel to the equatorial plane, a horizontal sundial can be thought as the projection of it on the horizontal plane. As can be seen in Figure 1, the stick of the equatorial sundial, which is parallel to the universe axis (polos) intersects the equatorial sundial at the point $M$. The radial $MF$ hour lines that have $15^\circ$ angle difference from each other are around this point. 

A $Zawâlî$ hour system is obtained if the hour lines begins from $ME$ meridian, whereas it is defined as ghurubi hour system if they begins from $MG$ line$^9$ (of sunrise) or from $MH$ line$^{10}$ (of sunset). The diagram in Figure 2 shows the change in the beginning of hours between $Zawâlî$ hour system and both of $ghurûbi$ hour systems throughout the year in Istanbul. Accordingly, while the $Zawâlî$ system shows the time remaining to the midday or the time elapsed after it, $ghurûbi$ system shows the time elapsed after sunrise or the time remaining to the sunset.

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9. This hour system originated from Mesopotamia is called Babylon hour system.

10. Due to the fact that the Italians first started to use this hour system, after having been introduced in Islamic World, it is called, in other European countries, the Italian hour system.
2.1. Deriving Equations of Hour Angles and Lines

If we turn the $ABCD$ plane that is parallel to equatorial plane in the Figure 1 through $A-B$ line, placing it on the $ABC'D'$ horizon, we get the position depicted in Figure 3. On that, the point $M'$ is the folded position of $M$, in which the sundial stick intersects the equatorial plane on the horizon. Point $O$ represents the point where the universe axis intersects the horizon, and $MEO$ right triangle represents noon plane lied over the horizon. Since the $ME$ edge of this triangle gives the direction of the universe axis, the $MEO$ angle is equal to the complementary of the latitude of the present location: $\angle (MEO) = (90° - \varphi)$. If we take any $\angle (EMF) = \angle (EM'F') = h$ hour angle into consideration, the hour line that corresponds to this angle on the horizon is $OF$, and the hour angle is $\angle (EOF) = s$. Yet, as a corollary to the Figure 3, the following is found:
\( \tan(h) = \frac{EF}{EM'} \)  
\( EO = \frac{EM}{\sin \varphi} = \frac{EM'}{(\sin \varphi)} \)  
\( \tan(s) = \frac{EF}{EO} = \frac{(EF)/(EM')}{(\sin \varphi)} = (\sin \varphi).\tan(h) \)  
\( \tan(s) = (\sin \varphi).\tan(h) \)

**Figure 3.** Angles of Hour lines and their lengths in horizontal sundial.

According to this, given the stick length \( p = MK \), hour angles \( s \) and lengths \( l = OF \) can be calculated as following:

\[
s = \tan^{-1} \left[ (\sin \varphi).\tan(h) \right]
\]

and since is \( EK = p.(\tan \varphi) \), \( KO = p/(\tan \varphi) \)

\[
l = OF = EO/(\cos s) = (EK + KO)/(\cos s)
\]

\[
= p.\{(\tan \varphi) + 1/(\tan \varphi)\}/(\cos s) = p/[(\cos s).\sin \varphi].\cos \varphi = l/p = 2/[(\cos s).\sin 2\varphi]
\]

is found. However, in *ruhame*, all hour angles and lengths are calculated with reference to the point \( K \) where \( MK \) stick is perpendicularly located to the horizon. As a result, \( t = \angle EKF \) hour angle and \( m = KF \) length need to be expressed in terms of other hour parameters. If sinus theorem is applied to the \( KOF \) triangle, since \( \sin(180° - t) = (\sin t) \),

\[
\frac{\sin t}{l} = \frac{(\sin s)}{m}
\]

this expression, together with (2) above and \( (\cos t) = (EK)/m = (p/m).\tan(\varphi) \)
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\[
(tan t) = (sin t)/(cos t) = [(l/m) (sin s)]/[(p/m)(tan \varphi)] = [(l/p)(sin s)]/(cos \varphi)
\]

\[
(tan \varphi) = \frac{(sin s)/(\cos s)(\sin \varphi)(\cos \varphi)(\tan \varphi)}{\cos \varphi} = (\tan s)/(\sin \varphi^2)
\]

\[
t = \tan^{-1}\left[\frac{\tan \varphi}{\sin \varphi^2}\right]
\]

is found. Taking EFK right triangle into consideration, since \(EF = m.(\sin t)\) and \(EK = p.(\tan \varphi)\)

\[
m^2 = \ EF^2 + EK^2 = m^2.(\sin t)^2 + p^2.(\tan \varphi)^2
\]

or

\[
m/p = (\tan \varphi)/(\cos t)
\]

can be written. Hence, since \(s\) and \(\varphi\) is known, \(t\) and \((m/p)\) can be calculated.

However, these expressions are valid only for hour lines' \(F = I\) points on the equinox line. Throughout the year, sun rays reaches to the earth with an angle of declination of \(\delta = \varepsilon = 23^\circ.5\). Two times a year, on March 21 (the beginning of Aries) and on September 21 (the beginning of Libra), the slope angle is \(\delta = 0\). In these dates, shade length is on the equinox line all day, and nights and days are equal. On June 21 (the beginning of Cancer) we have \(\delta = \varepsilon = 23^\circ.5\) and summer solstice is the longest day of the year. On the other hand, on December 21 (the beginning of Capricorn) we have \(\delta = \varepsilon = -23^\circ.5\) and winter solstice is the shortest day of the year. In all days except for equinoctial ones, the shade of the stick makes a hyperbolic curve on sundial's plane.

To draw the ruhāma, calculation of the shade points denoting \(I\), especially during June 21 and December 21 (beginning of Cancer and Capricorn, respectively) is necessary. For this, since we know that in the drawing projected onto the horizon plane, the shade falls on the \(ME\) line in equinox day, that line is lengthened, and since we know that in this situation the shade length is equal to \(OF\) line, we draw a circle with the center \(O\) and radius \(OF\), and point \(F'\) is found (Figure 4). On the stick plane that is lied on the horizon, \(OF'\) line is equal to hour line with respect to its lengths. If we take \(<(FMF)' = \varepsilon\) and \(<(FMF)' = \varepsilon\) angles and intersect them with \(OF'\) line, \(OF'_y = l'_y\) corresponds to shade length at the beginning of the Cancer, and \(OF'_o = l'_o\) corresponds to the shade length at the beginning of the Capricorn. Since the real position of these shades is on the \(OF\) hour line, by using the arcs with center \(O\) and radius \(OF'_y = l'_y\) and \(OF'_o = l'_o\), the real \(F'_y\) and \(F'_o\) points on \(OF\) can be obtained. Yet, angles and distances given in Ruhāma are \(t\) and \(s\) angles and, \(m_y\) and \(m_o\) distances defined with reference to \(K\) point.

To calculate these angles and distances, first the lengths of \(OF'_y = l'_y\) and \(OF'_o = l'_o\) should be calculated. Since in \(MF'O\) triangle, \(OF' = OF = I\), when sinus theorem is applied, \(<(MF'O) = w\) angle can be calculated:
\[ w = \sin^{-1}\left\{1/\left(\frac{l}{p}\right) (\sin \varphi)\right\} \]

Similarly, by taking exterior angles into account, it can be seen that for \( MF_{y}O \) triangle, \( \angle (MF'O) = w_y = (w + \varepsilon) \) and for \( MF_{o}O \) triangle \( \angle (MF'O) = w_o = (w - \varepsilon). \) Accordingly, if sinus theorem is applied to \( MF_{y}O \) and \( MF_{o}O \) triangles, respectively

\[ l_y = \frac{p. (\cos \varepsilon)}{\left(\sin \varphi \right) \left[ \sin (w + \varepsilon)\right]} \]
\[ l_o = \frac{p. (\cos \varepsilon)}{\left[ \sin \varphi \right] \left[ \sin (w - \varepsilon)\right]} \]

can be found. To calculate the K-centered angle and distances, first, sinus theorem should be calculated for \( \angle (KFO) = x \) in \( OFK \) triangle:

\[ x = \sin^{-1}\left\{\frac{p. (\sin s)}{\left[ m \tan \varphi \right]}\right\} \]

As a result, if the expression

\[ (\sin t_y) = \left(\frac{l}{p}\right) \left[ \sin (t_y + x - t)\right] \left[ \tan \varphi \right] \]

obtained from sinus theorem applied to \( OKF_y \) triangle is arranged,

\[ t_y = \tan^{-1}\left\{\frac{\left[ \sin (x - t)\right]}{\left[ \left(\cot \varphi \right) / \left(\frac{l_y}{p}\right)\right] - \left[ \cos (x - t)\right]}\right\} \]

and from sinus theorem applied to \( KFF_y \) triangle

\[ m_y / p = \left[ \left(\frac{l_y}{p}\right) (\sin s)\right] / \left(\sin t_y\right) \]

is calculated. Similarly when sinus theorem is applied to \( OKF_o \) triangle,

\[ (\sin t_o) = \left(\frac{l}{p}\right) \left[ \sin (x - t + t_o)\right] \left[ \tan \varphi \right] \]
\[ t_o = \tan^{-1}\left\{\frac{\left[ \sin (x - t)\right]}{\left[ \left(\cot \varphi \right) / \left(\frac{l_o}{p}\right)\right] - \left[ \cos (x - t)\right]}\right\} \]

can be found. And from the sinus theorem applied to \( KFF_o \) triangle

\[ m_o / p = \left[ \left(\frac{l_o}{p}\right) (\sin s)\right] / \left(\sin t_o\right) \]

can be calculated. \( t_y \) angles and \( m_y \) shade lengths indicate \( Y_i \) Cancer points, and \( t_o \) angles and \( m_o \) shade lengths indicate \( O_i \) Capricorn points.
Figure 4

All these calculated values are valid for zawālî hours. If we want them to be valid also for ghurūbî hours that take sunrise or sunset as starting point as is the case for ruhāma, it should be taken into account that in Istanbul in the beginning of Cancer, compared to the equinox point where day and night is equal, sunrise and sunsets are early and late, respectively, as the amount of half-day remnant $F$. In contrast, in the beginning of Capricorn, compared to the equinox point, in Istanbul, sunrise and sunset are late and early, respectively, as the amount of half-day remnant $F$. (Figure 5)

Figure 5
In solstices, considering $AB = 90^\circ - \varphi = 49^\circ$ and $BC = \varepsilon = 23^\circ.5$ in Istanbul, half-day remnant $F$, is calculated as $F = \sin^{-1} \left[ (\tan \varphi) \cdot (\tan \varepsilon) \right] = 22^\circ.2$. Accordingly, to calculate $Y$ and $O$ points, equation of daylight should be added to or subtracted from $h_i$ angles. The expressions for calculation of these points are equal to the ones used in calculating $I_i$ points.

**2.1.1. Drawing Hour Lines Elapsed after sunrise [Dā‘ir min al- shurūq] and Hour Lines remaining to sunset [Dā‘ir ilā al- ghurūb]**

For this, $Y_i$ points of the beginning of Cancer, and $O_i$ points of the beginning of Capricorn should be calculated. Yet, to find these, first, $I_i$ equinox points, then $I_{yi}$ Cancer and $I_{oi}$ Capricorn zawālī shade points, and then $I_{yyi}$ Cancer and $I_{ooy}$ Capricorn ghurubi shade points need to be calculated.

Using the expressions derived above, let us give which relevant relations are used to calculate the values that give the important points of a horizontal sundial that is to be used in Istanbul:

Since for equatorial sundials, each hour corresponds to $15^\circ$ degrees, after the sunrise, angles regarding each hour on the equinox line being as $i = 1,2,...6$

$$h_i = 90^\circ - i.15^\circ$$

should be taken. As Istanbul’s latitude is $\varphi = 41^\circ$, from the expression (1), horizontal hour angles can be found from

$$s_i = \tan^{-1} [0,656 \cdot (\tan h_i)]$$

and from the expression (2) shade lengths with reference to equinox points can be found from

$$l/p = (2.01966)/(\cos s_i)$$

When these values are calculated with reference to $K$ point where the stick stands, with the help of (3) and (4), for $t_i$ angle and $m/p$ shade lengths

$$t_i = \tan^{-1} [(2.32335) \cdot (\tan s_i)]$$

$$m/p = (0.86929)/(\cos t_i)$$

is obtained. Since in the work of Ruhāma, $t_i$ angles are taken from the west point with reference to south it is evaluated as $(90^\circ - t_i)$. Table 1 shows $t_i$ angles of $I_i$ shade points on the equinox line and $m_i/p$ distances.
Since for \( I_7, \ldots, I_{11} \) points, which are on the east side of the ruhāma, sundial drawing is symmetrical with reference to north-south line, \( t_i \) angles are taken with reference to west point.

To calculate \( Y_i \) and \( O_i \) points, which are time after sunrise and time to sunset, respectively, the equation of daylight with the degree of 22°.2 needs to be added to or subtracted from \( h_i \) angles. Other expressions can be used same as in the calculation of \( I_i \) points. Table 2 shows the comparison of calculated and given points.

### Table 2

<table>
<thead>
<tr>
<th>( i )</th>
<th>( Y_i/O_i )</th>
<th>( h_i )</th>
<th>( (90° - t_p) )</th>
<th>( m_p/p )</th>
<th>( (90° - t_m) )</th>
<th>( m_m/p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( Y_1/O_1 )</td>
<td>97°.2/52°.8</td>
<td>-22°/-22° 46'</td>
<td>5°.8090</td>
<td>80°.0289</td>
<td>09° 58'16''</td>
</tr>
<tr>
<td>2</td>
<td>( Y_2/O_2 )</td>
<td>82°.2/37°.8</td>
<td>-13°/-13° 32'</td>
<td>3°.0570</td>
<td>69°.2528</td>
<td>20° 44'50''</td>
</tr>
<tr>
<td>3</td>
<td>( Y_3/O_3 )</td>
<td>67°.2/22°.8</td>
<td>-04°/-05° 31'</td>
<td>2°.4154</td>
<td>56°.7304</td>
<td>56° 43'49''</td>
</tr>
<tr>
<td>4</td>
<td>( Y_4/O_4 )</td>
<td>52°.2/07°.8</td>
<td>6°/ 4°</td>
<td>2°.1597</td>
<td>41°.3462</td>
<td>48° 39'14''</td>
</tr>
<tr>
<td>5</td>
<td>( Y_5/O_5 )</td>
<td>37°.2/0°</td>
<td>20°/ 15°</td>
<td>2°.0506</td>
<td>22°.2144</td>
<td>67° 47'44''</td>
</tr>
<tr>
<td>6</td>
<td>( Y_6/O_6 )</td>
<td>22°.2/0°</td>
<td>36°/ 25° 12'</td>
<td>2°.0197</td>
<td>0°</td>
<td>90°</td>
</tr>
<tr>
<td>7</td>
<td>( Y_7/O_7 )</td>
<td>7°.2/0°</td>
<td>0°</td>
<td>0°</td>
<td></td>
<td></td>
</tr>
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<td>( Y_8/O_8 )</td>
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<td>0°</td>
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</tr>
<tr>
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<td>0°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>( Y_{10}/O_{10} )</td>
<td>0°</td>
<td>0°</td>
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### 2.1.2. Drawing the Tropics of Cancer and Libra

The angle and lengths of the shade falling upon the equinox line for Ariel and Libra can be obtained by the line connecting \( I_1 = I_{11} \) points. The calculated angle \( (90° - t_j) = (90° - t_{j1}) = 09° 58' \) and shade length \( m_j = m_{j1} = 12.(5° 01' 44'') \) of these two points are equal to the one given in the text. (See Table 1)
2.1.3 Drawing Hour lines to Midday and Hours after Midday

[Fażl al-dā’īr]

To draw the hour lines for hours to and after midday, beginning points of $I_{yi}$ Sagittarius and $I_{oi}$ Capricorn which corresponds to $I_{i}$ equinox points that are formulated with reference to $K$ points given above need to be calculated. From the expressions given above, these relations can be calculated as follows:

\[ w_{i} = \sin^{-1} \left[ (1,5243)/(l/p) \right] \]
\[ l_{y}/p = (1,3978)/[\sin (w + 23.5^\circ)] \]
\[ l_{o}/p = (1,3978)/[\sin (w - 23.5^\circ)] \]
\[ x = \sin^{-1} \left[ ((1,1504).\sin(s_{j}))/((m/p)) \right] \]
\[ t_{yi} = \tan^{-1} \left[ \frac{\sin(x_{i} - t_{j})}{[1/((0,869)/(l_{y}/p))] - \cos(x_{i} - t_{j})} \right] \]
\[ t_{oi} = \tan^{-1} \left[ \frac{\sin(x_{i} - t_{j})}{[1/((0,869)/(l_{o}/p))] - \cos(x_{i} - t_{j})} \right] \]
\[ m_{yi}/p = [(l_{y}/p).\sin(s_{j})]/(\sin t_{y}) \]
\[ m_{oi}/p = [(l_{o}/p).\sin(s_{j})]/(\sin t_{o}) \]

The values regarding the shade angles $(90^\circ - t_{yi})$ and $(90^\circ - t_{oi})$ of $I_{yi}$ and $I_{oi}$ points that are evaluated, and the shade lengths of $m_{yi}/(6-i)$ and $m_{oi}/(6-i)$ are compared to those given in the text in Table 3:

Table 3

<table>
<thead>
<tr>
<th>Hour</th>
<th>Hours to Midday</th>
<th>Hours after Midday</th>
<th>$(90^\circ - t_{yi(i))/ (90^\circ - t_{oi(i)})$</th>
<th>$m_{yi(6-i)}, m_{oi(6-i)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$I_{yi}-I_{yi1}$</td>
<td>$I_{yi2}-I_{yi3}$</td>
<td>$-18^\circ 10'/ -13^\circ$</td>
<td>$18^\circ 10'/ -13^\circ$, $44^\circ 17'/ 62^\circ$, $44^\circ 17'/ 62^\circ$</td>
</tr>
<tr>
<td>5</td>
<td>$I_{yi1}-I_{yi1}$</td>
<td>$I_{yi2}-I_{yi1}$</td>
<td>$-08^\circ 19'/ 09^\circ 58'$</td>
<td>$-12^\circ / -$, $24^\circ / 29^\circ 27'$, $25^\circ 15'/ -$</td>
</tr>
<tr>
<td>4</td>
<td>$I_{yi3}(L)-I_{yi2}$</td>
<td>$I_{yi4}(L)-I_{yi3}$</td>
<td>$-00^\circ 09'/ 37^\circ 09'$</td>
<td>$8^\circ 22'/ -$, $15^\circ 41'/ -$, $14^\circ / -$</td>
</tr>
<tr>
<td>3</td>
<td>$I_{yi1}(L)-I_{yia}$</td>
<td>$I_{yi0}(L)-I_{yia}$</td>
<td>$10^\circ 52'/ 48^\circ 15'$</td>
<td>$10^\circ /$, $10^\circ 33'/ -$</td>
</tr>
<tr>
<td>2</td>
<td>$I_{yi5}(L)-I_{yi4}$</td>
<td>$I_{yi6}(L)-I_{yi5}$</td>
<td>$25^\circ 65'/ 60^\circ 51'$</td>
<td>$25^\circ /$, $7^\circ 05'/ -$, $6^\circ / -$</td>
</tr>
<tr>
<td>1</td>
<td>$I_{yi7}(L)-I_{yi6}$</td>
<td>$I_{yi8}(L)-I_{yi7}$</td>
<td>$49^\circ 44'/ 74^\circ 56'$</td>
<td>$49^\circ 44'/ -$, $4^\circ 40'/ -$, $58^\circ 45'/ -$</td>
</tr>
<tr>
<td>0</td>
<td>$I_{yi9}(L)-I_{yia}$</td>
<td></td>
<td>$90^\circ$</td>
<td>$3^\circ 47'/ -$</td>
</tr>
</tbody>
</table>

2.1.4 Drawing the Curves of Asr-i Awwal and Asr-i Thāni

In ruhame, there are two curves for asr prayer time defined as asr-i awwal and asr-i thāni. If the length of sundial stick $p$ and the highest point of the Sun $m_{6} = (a-b)$ are given, the shade length for asr-i awwal in that day can be calculated by $a_{1} = (b-d) = (m_{6} + p)$, and that of asr-i thāni can be calculated by $a_{2} = (b-e) = (m_{6} + 2.p)$ (Figure 6). If the distances are proportioned to the stick length $p$, as is the case in ruhama, the following expressions can be obtained:
\[ a_1/p = (m_y/p + 1) \]
\[ a_2/p = (m_y/p + 2). \]

Since the midday shades defined with reference to the point \( K \) are already calculated for equinox and tropics, it is easy to find \( a_1/p \) and \( a_2/p \) distances. Yet, to find the \( t_1 \) and \( t_2 \) angles, the expressions given above should be calculated in accordance to these distances.

![Figure 6 Definitions of the Time of Asr-i Awwal and Asr-i Thani](image)

Table 4 shows the calculated and shade lengths given in the text, for the points of \( asr-i\ awwal \) and \( asr-i\ thani \). Some of the values given in the original text, which are marked with red in the table are incorrect. They could be the result of incorrect copying (\( istinsakh \)).

For the \( A_1 \) points on the equinox line, from the expression (4)

\[ t_i = \cos^{-1}\left(\frac{(\tan \varphi)/(a/p)}\right), \]

The easiest way to determine \( A_{1y} \) and \( A_{1o} \) points that are at the beginning of Cancer and Capricorn is determining \( taiy \) and \( taio \) angles on the drawing of \( ruhame \) by drawing circles with \( K \) as the center and \( aiy \) and \( aio \) as radiuses, and by guessing their intersection points with the hyperbolic curve. Otherwise, hyperbolic equations regarding Cancer and Capricorn need to be found and intersected with the aforesaid circle. Table 5 summarizes the results regarding \( taiy \) and \( taio \) angles.

<table>
<thead>
<tr>
<th>Point</th>
<th>Definition</th>
<th>Calculated</th>
<th>Given</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{1y} )</td>
<td>( a_{1y}/p = m_{y}/p + 1 )</td>
<td>1°.32</td>
<td>15° 47'</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>( a_{1}/p = m_{y}/p + 1 )</td>
<td>1°.87</td>
<td>22° 26'</td>
</tr>
<tr>
<td>( A_{1o} )</td>
<td>( a_{1o}/p = m_{o}/p + 1 )</td>
<td>3°.10</td>
<td>37° 35'</td>
</tr>
<tr>
<td>( A_{2y} )</td>
<td>( a_{2y}/p = m_{y}/p + 2 )</td>
<td>2°.32</td>
<td>27° 47'</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( a_{2}/p = m_{y}/p + 2 )</td>
<td>2°.87</td>
<td>34° 26'</td>
</tr>
<tr>
<td>( A_{2o} )</td>
<td>( a_{2o}/p = m_{o}/p + 2 )</td>
<td>4°.10</td>
<td>49° 10'</td>
</tr>
</tbody>
</table>
Table 5

<table>
<thead>
<tr>
<th>Point</th>
<th>Expression</th>
<th>Calculated or Measured</th>
<th>Given</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$t_{ai}$</td>
<td>$90^\circ - t_{ai}$</td>
<td>$90^\circ - t_{ai}$</td>
</tr>
<tr>
<td>$A_{1y}$</td>
<td>-</td>
<td>$0^\circ 10'$</td>
<td>$0^\circ 10'$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>$t_{ai} = \cos^{-1}[(\tan \varphi)/(a_i/p)]$</td>
<td>$27^\circ.70 = 27^\circ 42'$</td>
<td>$27^\circ 48'$</td>
</tr>
<tr>
<td>$A_{1x}$</td>
<td>-</td>
<td>$57^\circ$</td>
<td>$56^\circ 48'$</td>
</tr>
<tr>
<td>$A_{2y}$</td>
<td>-</td>
<td>$-11^\circ$</td>
<td>$-11^\circ 32'$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$t_{ai} = \cos^{-1}[(\tan \varphi)/(a_i/p)]$</td>
<td>$17^\circ.63 = 17^\circ 38'$</td>
<td>$09^\circ 26'$</td>
</tr>
<tr>
<td>$A_{2x}$</td>
<td>-</td>
<td>$49^\circ$</td>
<td>$49^\circ 10'$</td>
</tr>
</tbody>
</table>

Conclusion

Based upon the results obtained above, if a table of angles and distances of hour lines is prepared without any mistake, it is very easy to practically draw a horizontal sundial or *ruhāma* for Istanbul. Since in the drawing the author added to the end of the text, that the north-south line or midday line is drawn with a slope of 45 degrees to the right side of the paper, it gives a false impression to the reader that it is a vertical sundial (*Picture 3*). Even in vertical sundials east-west line is drawn parallel to the lower edge of paper. On the other hand, in horizontal sundials, equinox line is usually drawn parallel to the lower edge of paper. Given that, it is still a mystery how the author calculated these values and wrote the text.

*Picture 3* Sundial drawing at the end of the text.

In the original text, $10^\circ$ is written instead of $10'$ by mistake.
Bibliography


Bursalı Mehmet Tahir Bey, Sicill-i-Osmāni, cilt 5, İstanbul: Tarih Vakfı Yurt Yayınları, 1996.


King, D. A., "Mizwala", EI2, c. 7, 115-16.


Risālah-i vaż′-i ruhāma .


Sâbiş b. Kurrā, Kitāb fi ālāt al- sa′āh allatì tusammā ruhāmāt, Köprülũ Kütüphanesi (İstanbul), No: 984/1, (Süleymaniye kütüphanesi, dijital arşiv no: 1435)
Appendix

**Figure A:** Board used in drawing ruhāma

**Figure B:** Drawing scale used in drawing ruhāma

**Figure C:** Points determined in drawing sundial on ruhāma
Facsimile of Ruhāma
Atilla Bir, Şinasi Acar, Mustafa Kaçar, A Mathematical Analysis of the Theory of Horizontal Sundials in the Ottoman Period: The Case of Risalah of Ruhame