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A Mathematical Analysis of the Theory of Horizontal Sundials in the Ottoman Period: The Case of *Risālah of Ruhāma*

Atilla Bir,* Şinasi Acar,** Mustafa Kaçar***

Abstract: As the prayer times are determined according the sun's position, the theory of sundials, how to draw and design them, hold a significant place in the history of Islamic astronomy. All Muslim scientists working on applied and theoretical sciences have books on this subject. Besides, as it was a part of *madrasa* curriculum, there are considerably high number of unexamined books from the beginning to the end of the Ottoman period. This study is about a manuscript titled *Risālah-i Vaż'-i Ruhāma*, that we received from a second-hand book seller. Its anonymous author argues that he offers a simpler way to draw sundials designed for Istanbul. The author leaves the person willing to make a sundial free to determine the height of the gnomon and he shows a practical method to draw ruhāma with the help of goniometer and ruler prepared according to the determined height. Although the article presents the mathematical bases of the method, it is unable to verify clearly how the author derived the numeric values given in the book. To the end of our study, facsimile text and its simplified transcription are attached.

Keywords: Theory of Sundials, gnomonic, Muslim Sundials, horizontal sundials, time systems in Islam, prayer times, *ruhāma*.

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1. Introductio

The time period of the Sun virtually revolving around the world is defined as 1 *full day*. In ancient Egypt – without taking the seasons into account – the days and nights, independent of each other, are divided into 12 equal hours. This conception of time is labeled as *seasonal* or *temporal* hour. On the 41st latitude, where Istanbul is located, on the 22nd of June, the longest day is 16 hours and 22 minutes, and the shortest night is 7 hours and 38 minutes; and on the 22nd of December, the shortest day is 9 hours and 27 minutes, whereas the longest night is 14 hours and 33 minutes. If these time lengths are divided into 12, in the summers the longest day hour is 1 hour and 21.83 minutes, and the shortest night hour is 38.17 minutes; and in the winters the shortest day hour is 47.25 minutes, and the longest night hour is 1 hour and 12.75 minutes. On the 21st of March in the spring, and 23rd of September in the autumn, on the other hand, the time lengths of day and night are equal and each is 12 hours.[1]

In ancient Egypt, and in the ancient world, time was determined during the daylight by measuring the shade-length of a stick (gnomon) fixed into the ground. For example, the obelisk in Sultan Ahmet Square in Istanbul, which was brought to the city in the 4th century from the Temple of Ammon in Karnak/Egypt, was a gnomon of a sundial. The hours in the night used to be determined by the rising times of certain stars or by their passage through the longitude. In addition, sandglasses and water clocks were also used as auxiliary methods to supplement day time calculation. In sandglasses, time is measured by the flow of sand in a bottle, and in water clocks, the flow of the water quantity is measured in specific time intervals. Additionally, people used candle clocks at night.

In Mesopotamia, days are divided into 12 hours of equal length. Herodotus (490-425 BC) narrates that the Greeks learned about sundials and dividing days into 12 from Babylon. In the Hellenistic period (330 BC -30), days began to be divided into 24 equal hours, as it is the case today, and especially as it is like seasonal hours of March 21st and September 23rd. Such kind of hours are called equal hours. Yet, taking midday or midnight as the beginning of the new calendar day (*alafranga*, *zawālī*), and taking sunset as the beginning of new day (*alaturka*, *ezānī*) are two different applications of this time measurement.

In the Lunar calendar or Hijri calendar, Lunar months begin with the sighting of the Moon as a thin crescent during sunset. The new month begins the new cal-

1 Atilla Bir, Mustafa Kaçar, Şinasi Acar, *Güneş Saatleri Yapım Kılavuzu* (İstanbul: Biryıl Yayınları, 2010), p. 28.

endar day, and when the Sun's top edge is tangent to the horizon during sunset, the hour is accepted to be as 12:00 or 0:00. The night that begins is considered not to be part of the previous day but the new one. As a result of this relatively defined day, the time span between two consecutive nights is not exactly equal to 24 hours. In the springs, it is a little longer, and in the autumns it is a little shorter. Hence, mechanical clocks need to be set to 12 manually every day during sunset.

The moment when the center of the Sun is just on the circle of the longitude is defined as midday. Timepieces which accept that moment as 12:00 are called noon-related *alafranga* or *zawālī* hour times. The word *zawāl* means exact midday time. To prevent possible confusions that may stem from setting the beginning of a new day at midday, usually in this hour system - as it is the case today - the day begins 12 hours before the midday, which means midnight.[2]

Since fasting and five daily prayers are ordered by Islam, and since these deeds are closely related to specific time slots, Muslims paid considerable attention to determining the times of noon, mid- afternoon and sunrise. In addition to this characteristic which we do not observe in religions other than Islam, accumulation of experience about this subject led to the production of more exact and more accurate timepieces, which in the past used to be produced just to know the time of the day. Besides, we do not know use of any devices for determining time, by Arabs in the pre-Islamic era, neither do we know their presence at the time of the Prophet Muhammad and four caliphs who succeeded him.

The first works about horizontal sundials are written in the Abbasid period. Although sources they used for calculating and drawing sundials were from pre-Islamic civilizations such as Ancient Greek, Egyptian and Indian civilizations, Muslim astronomers made considerable theoretical and practical contributions to the development of sundials. First Muslim astronomers who wrote on sundials are Ibrāhīm al-Fazārī (2nd/8th century), Habash al- Ḥāsib (3rd/9th century), Muḥammad ibn Mūsā al-Khwārizmī (2nd-3rd/8-9th century), Muḥammad ibn Ṣabbāh (3rd/9th century), al-Farghānī (3rd/9th century), Ibn al- ʻAdamī (3rd/9th century) and Abū ʻAbd Allah Muḥammad ibn al-Ḥasan ibn Abī Hishām al- Shatawī (3rd/9-10th century).[3] To our knowledge, the first book written in Islamic civilization about sundials is the al- Fazārī's *Kitāb al- mikyās li al- zawāl*, which he wrote when he was in Baghdad, but unfortunately is lost.[4] The oldest extant book on Islamic sundial is *ʻAmal al- sāʻa fī*

2 Bir, Kaçar, Acar, *Güneş Saatleri Yapım Kılavuzu*, pp. 29-30.

3 D. A. King, "Mizwala", *EI*$_2$, c. 7, 115-16; D. A. King, *Astronomy in the Service of Islam* (Londra: Variorum Reprints, 1993), c. VIII; D. A. King, "Astronomical Instrumentation in the Medieval Near East", *Islamic Astronomical Instruments* içinde (Londra: Variorum Reprints, 1987), c. I.

4 Boris A. Rosenfeld, "Sundails in Islam", *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, ed. H. Selin (Boston & London: Kluwer Academic Pub. Dordrect), pp. 921- 922.

basīṭ al- ruhāma, which is written in the early 9th century in Baghdad and attributed to al-Khwārizmī. The most important work which clearly explains how different sundials known in the Islamic world are produced is Thābit ibn Qurra's *Kitāb fī ālāt al- sāʿa allatī tusammā ruhāmāt*.[5] The book is at the Koprulu Library in Istanbul.[6]

Picture 1. The cover page of Thābit ibn Qurra's *Kitāb fī ālāt al-sāʿa allatī tusammā ruhāmāt* (Koprulu Library 984/1)

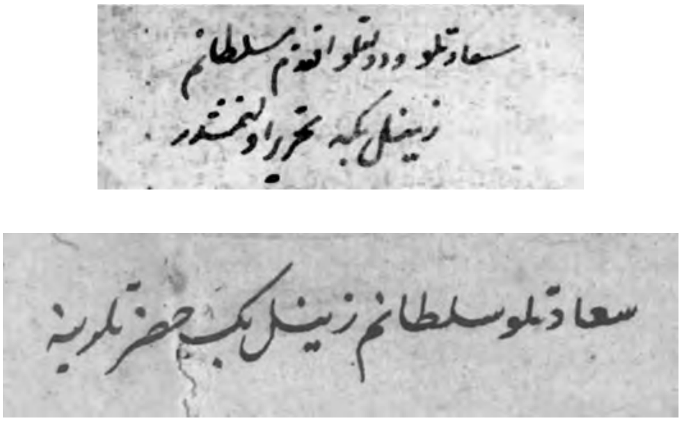
The manuscript entitled *Risālah-i vażʿ-i ruhāma* (*Book on Ruhame Construction*) that we analyze in this article is the first known work written in the Ottoman period, which discuss Islamic sundials. Its author is unknown. On the inside of its cover the following phrase is seen: "Written for his Excellency Zeynel Beg" (Picture 2-a). Zeynel Beg (d. 1589) is the grandson of Asad al- dīn Kalanī who founded the Hakkari Emirate. He was appointed as the Amir of Hakkari by the Ottoman state, and supported by Süleyman I (1520-1566), Selim II (1566-1574) and Murad III (1574-1595). He was martyred in Merent during the Tabriz campaign; and after the conquest of the city, his corpse was brought to Hakkari (it was called Çölemerik at the time), and buried in the courtyard of Zeynel Beg Madrasa which was built by him.[7] There is a very similar note "to his excellency Zeynel Beg", which gives the

5 Regis Morelon, "Thabit b. Qurra and Arab Astronomy in the 9th Century", *Arabic Sciences and Philosophy, A historical journal*, ed. R. Rashed, c. 4 (Cambridge, 1994), 111-139; Pouyan Rezvani, *Three Treatises by Thābit ibn Qurrah* (Tahran, 2013), pp. 3-4.

6 Köprülü Library, nr. 984/1.

7 "Zeynel Beg was a loyal and brave one among Kurdish Begs in Hakkari [a city located in southeastern

impression of being written by the same person, in a corpus (majmuʻa) at Topkapi Palace Library, in the collection of Hazine (n. 452), which includes writings on astronomy and astrology about Taqī al- dīn al- Rāṣid and Aḥmad-i Dāʻī (Picture 2-b). The 17.5x11 cm sized hard-covered book that contains also this work includes three different works. The first one is an astronomy book about stars, and the third one is an astrology book dated 890/1485.



Picture 2 (a) The phrase from the inside of the cover of the manuscript which states it is written for Zeynel Bag (above), (b) A similar dedication phrase from the manuscript at Topkapı Palace Library Hazine number 452 (below).

Though in the manuscript it is argued that drawing sundial is very easy and absolutely accurate, the text does not mention the kind of sundial. While it is stated that the sundial is designed for Istanbul ($\varphi = 41°$), the text does not give information about the azimuth angle of the wall which is required for the vertical sundials. The fact that the values about both kind of *ghurūbī* hour systems (*Babylonian* and *italic*) that are used in the drawing, with *zawālī* hour system, are given symmetric to the meridian, and more especially that the equinoctial line is given horizontal indicate that this timepiece is a horizontal sundial. In addition, the fact that drawing the *asr-i awwal* and *asr-i thāni* curves on the same plane with both *ghurūbī* hour system supports this assumption. Since wall clocks are usually drawn directly onto the wall, the name *ruhāma,* literally means marble, given to this timepiece should refer to a horizontal sundial.[8]

part of Turkey], who lived in the time of Süleyman I." (*Sicilli-Osmani* 5, Tarih Vakfı Yurt Yayınları, 1996, 1709). (translation made by the authors)

8 There is a sundail drawing at the end of the text, but there is nothing on the drawing which makes reference to the text itself. We will present our opinions about that drawing at the end of the article.

2. Mathematical Analysis

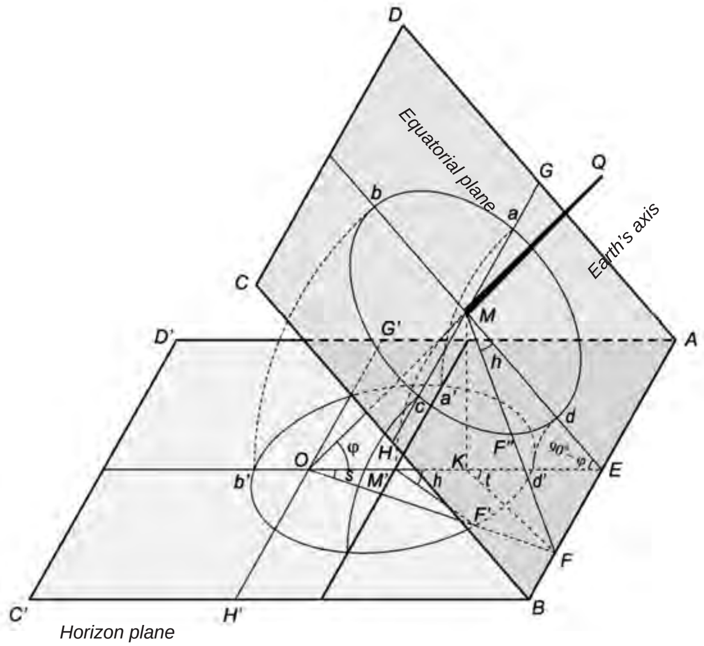


Figure 1. The relation between equatorial sundial and horizontal sundial

Since the quadrant of an equatorial sundial is parallel to the equatorial plane, a horizontal sundial can be thought as the projection of it on the horizontal plane. As can be seen in Figure 1, the stick of the equatorial sundial, which is parallel to the universe axis (polos) intersects the equatorial sundial at the point *M*. The radial *MF hour lines* that have 15° angle difference from each other are around this point. *Zawālī* hour system is obtained if the hour lines begins from *ME meridian*, whereas it is defined as ghurubi hour system if they begins from *MG line*[9] (of sunrise) or from *MH line*[10] (of sunset). The diagram in Figure 2 shows the change in the beginning of hours between *zawalī* hour system and both of *ghurūbī* hour systems throughout the year in Istanbul. Accordingly, while the *zawālī* system shows the time remaining to the midday or the time elapsed after it, *ghurūbī* system shows the time elapsed after sunrise or the time remaining to the sunset.

9 This hour system originated from Mesopotamia is called Babylon hour system.

10 Due to the fact that the Italiens first started to use this hour system, after having been introduced in Islamic World, it is called, in other European countries, the Italic hour system.

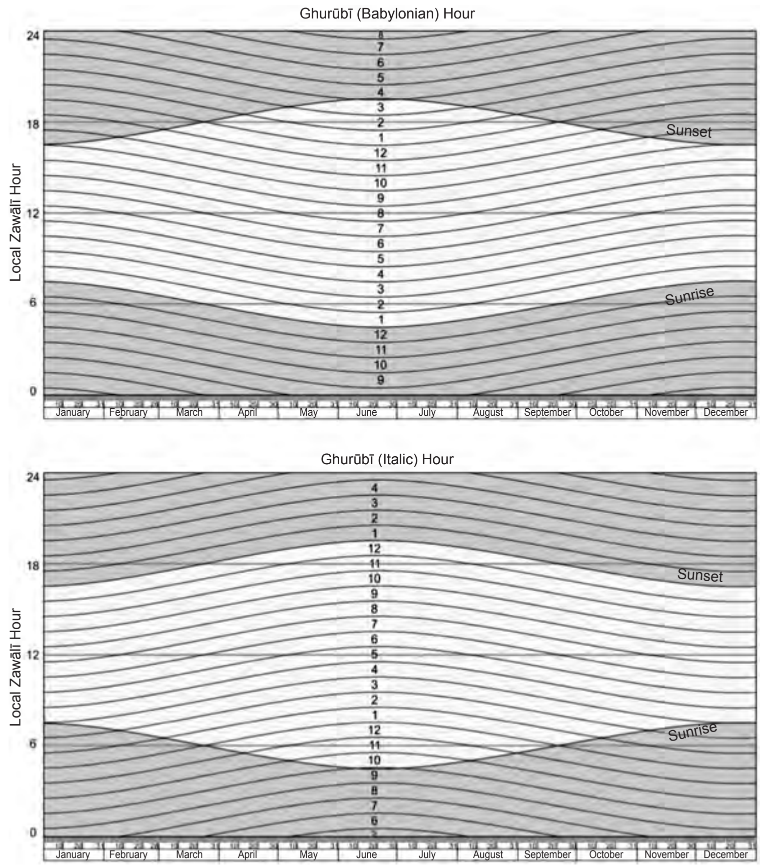


Figure 2. Diagrams for the change between *zawālī* and *ghurūbī* hour systems in a year on the latitude of Istanbul.

2.1. Deriving Equations of Hour Angles and Lines

If we turn the *ABCD* plane that is parallel to equatorial plane in the Figure 1 through *A-B line*, placing it on the *ABC'D'* horizon, we get the position depicted in Figure 3. On that, the point *M'* is the folded position of *M,* in which the sundial stick intersects the equatorial plane on the horizon. Point *O* represents the point where the universe axis intersects the horizon, and *MEO* right triangle represents noon plane lied over the horizon. Since the *ME* edge of this triangle gives the direction of the universe axis, the *MEO* angle is equal to the complementary of the latitude of the present location: < (*MEO*) = (90° - φ). If we take any < (*EMF*) = <(*EM'F'*) = h hour angle into consideration, the hour line that corresponds to this angle on the horizon is *OF,* and the hour angle is < (*EOF*) = *s*. Yet, as a corollary to the Figure 3, the following is found:

$(\tan h) = (EF)/(EM')$

$(EO) = (EM)/(\sin \varphi) = (EM')/(\sin \varphi)$

$(\tan s) = (EF)/(EO) = [(EF)/(EM')].(\sin \varphi) = (\sin \varphi).(\tan h)$

$(\tan s) = (\sin \varphi).(\tan h)$

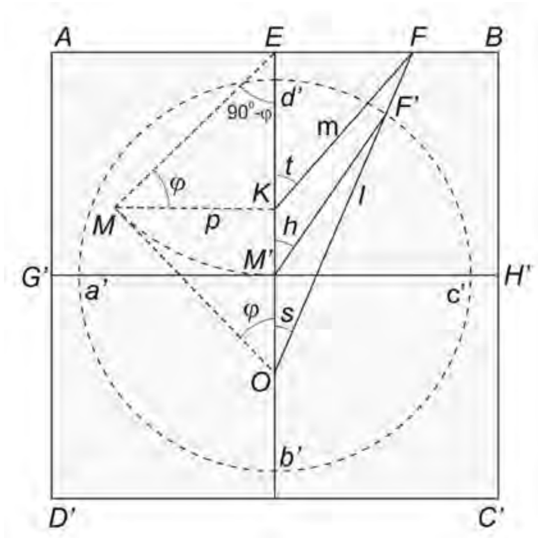


Figure 3. Angles of Hour lines and their lengths in horizontal sundial.

According to this, given the stick length $p = MK$, hour angles s and lengths $l = OF$ can be calculated as following:

$s = \tan^{-1} [(\sin \varphi).(\tan h)]$ (1)

and since is $EK = p.(\tan \varphi)$, $KO = p/(\tan \varphi)$

$l = OF = EO/(\cos s) = (EK + KO)/(\cos s)$

$= p.[(\tan \varphi) + 1/(\tan \varphi)]/(\cos s) = p/[(\cos s).(\sin \varphi).(\cos \varphi)]$

$l/p = 2/[(\cos s).(\sin 2\varphi)]$ (2)

is found. However, in *ruhame*, all hour angles and lengths are calculated with reference to the point K where MK stick is perpendicularly located to the horizon. As a result, $t = < EKF$ hour angle and $m = KF$ length need to be expressed in terms of other hour parameters. If sinus theorem is applied to the KOF triangle, since $[\sin (180° - t)] = (\sin t)$,

$(\sin t)/l = (\sin s)/m$

this expression, together with (2) above and $(\cos t) = (EK)/m = (p/m).(\tan \varphi)$

(tan t) = (sin t)/(cos t) = [(l/m) (sin s)]/[(p/m).(tan φ)]= [(l/p).(sin s)]/(tan φ)

= (sin s)/[(cos s).(sin φ).(cos φ).(tan φ)] = (tan s)/(sin φ)^2

$t = \tan^{-1}[(\tan s)/(\sin \varphi)^2]$ (3)

is found. Taking *EFK* right triangle into consideration, since $EF = m$.(sin t) and $EK = p$.(tan φ)

$m^2 = EF^2 + EK^2 = m^2.(\sin t)^2 + p^2.(\tan \varphi)^2$

or

$m/p = (\tan \varphi)/(\cos t)$ (4)

can be written. Hence, since s and φ is known, t and (m/p) can be calculated.

However, these expressions are valid only for hour lines' $F \equiv I_i$ points on the equinox line. Throughout the year, sun rays reaches to the earth with an angle of declination of $\delta \leq \varepsilon = 23°.5$. Two times a year, on March 21 (the beginning of Aries) and on September 21 (the beginning of Libra), the slope angle is $\delta = 0$. In these dates, shade length is on the equinox line all day, and nights and days are equal. On June 21 (the beginning of Cancer) we have $\delta = \varepsilon = 23°.5$ and summer solstice is the longest day of the year. On the other hand, on December 21 (the beginning of Capricorn) we have $\delta = \varepsilon = -23°.5$ and winter solstice is the shortest day of the year. In all days except for equinoctial ones, the shade of the stick makes a hyperbolic curve on sundial's plane.

To draw the *ruhāma*, calculation of the shade points denoting I_i especially during June 21 and December 21 (beginning of Cancer and Capricorn, respectively) is necessary. For this, since we know that in the drawing projected onto the horizon plane, the shade falls on the *ME* line in equinox day, that line is lengthened, and since we know that in this situation the shade length is equal to *OF* line, we draw a circle with the center *O* and radius *OF,* and point *F'* is found (Figure 4). On the stick plane that is lied on the horizon, *OF'* line is equal to hour line with respect to its lenghts. If we take < ($F'MF_y'$) = ε and < ($F'MF_o'$) = ε angles and intersect them with *OF'* line, $OF'_y = l_y'$ corresponds to shade length at the beginning of the Cancer, and $OF_o' = = l_o'$ corresponds to the shade length at the beginning of the Capricorn. Since the real position of these shades is on the *OF* hour line, by using the arcs with center *O* and radius $OF_y = l_y$ and $OF_o = l_o$, the real F_y and F_o points on *OF* can be obtained. Yet, angles and distances given in *Ruhāma* are t_y and t_o angles and, m_y and m_o distances defined with reference to *K* point.

To calculate these angles and distances, first the lengths of $OF_y = l_y$ and $OF_o = l_o$ should be calculated. Since in *MF'O* triangle, $OF' = OF = l$, when sinus theorem is applied, < (*MF'O*) = w angle can be calculated:

$w = \sin^{-1} \{1/[(l/p)\,(\sin \varphi)]\}$

Similarly, by taking exterior angles into account, it can be seen that for $MF_y'O$ triangle, < (MF'O)= $w_y = (w + \varepsilon)$ and for $MF_o'O$ triangle < $(MF_o'O) = w_o = (w - \varepsilon)$. Accordingly, if sinus theorem is applied to $MF_y'O$ and $MF_o'O$ triangles, respectively

$l_y = \{p.(\cos \varepsilon)\}/\{(\sin \varphi).[\sin (w + \varepsilon)]\}$

$l_o = \{p.(\cos \varepsilon)\}/\{(\sin \varphi).[\sin (w - \varepsilon)]\}$

can be found. To calculate the K-centered angle and distances, first, sinus theorem should be calculated for < $(KFO)= x$ in OFK triangle:

$x = \sin^{-1} \{[p.(\sin s)]/[m\,(\tan \varphi)]\}$

As a result, if the expression

$(\sin t_y) = (l/p).[\sin (t_y + x - t)].(\tan \varphi)$ obtained from sinus theorem applied to OKF_y triangle is arranged,

$t_y = \tan^{-1} \{[\sin (x - t)]/\{[(\cot \varphi)/(l_y/p)] - [\cos (x - t)]\}\}$

and from sinus theorem applied to KFF_y triangle

$m_y/p = [(l_y/p).(\sin s)]/(\sin t_y)$

is calculated. Similarly when sinus theorem is applied to OKF_o triangle,

$(\sin t_o) = (l_o/p).[\sin (x - t + t_o)].(\tan \varphi)$

$t_o = \tan^{-1} \{[\sin (x - t)]/\{[(\cot \varphi)/(l_o/p)] - [\cos (x - t)]\}\}$

can be found. And from the sinus theorem applied to KFF_o triangle

$m_o/p = [(l_o/p).(\sin s)]/(\sin t_o)$

can be calculated. t_y angles and m_y shade lengths indicate Y_i Cancer points, and t_o angles and m_o shade lengths indicate O_i Capricorn points.

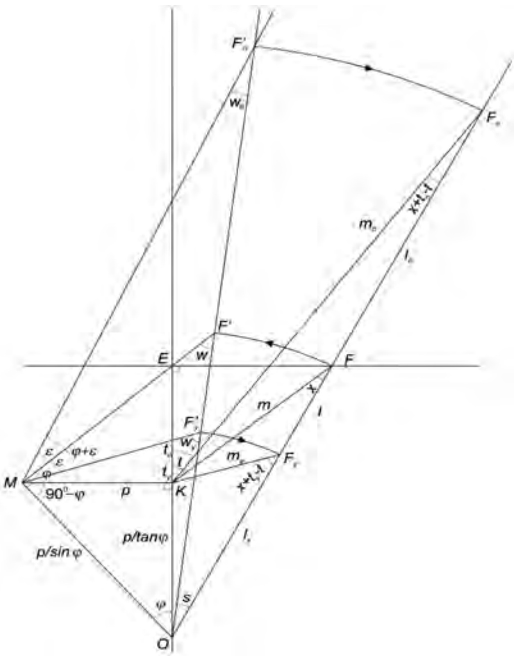


Figure 4

All these calculated values are valid for *zawālī* hours. If we want them to be valid also for *ghurūbī* hours that take sunrise or sunset as starting point as is the case for *ruhāma*, it should be taken into account that in Istanbul in the beginning of Cancer, compared to the equinox point where day and night is equal, sunrise and sunsets are early and late, respectively, as the amount of half-day remnant *F*. In contrast, in the beginning of Capricorn, compared to the equinox point, in Istanbul, sunrise and sunset are late and early, respectively, as the amount of half-day remnant *F*. (Figure 5)

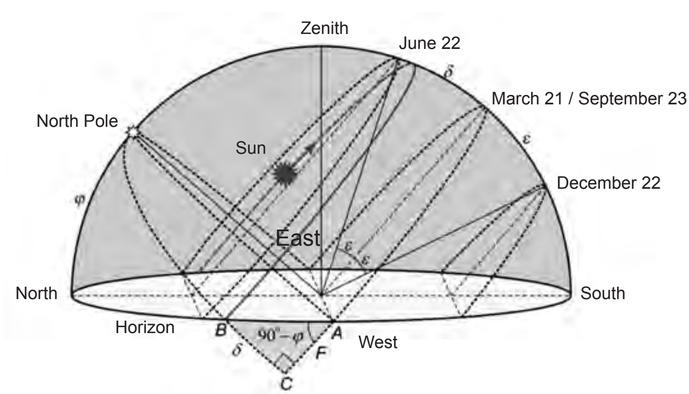


Figure 5

In solstices, considering $AB = 90° - \varphi = 49°$ ve $BC = \varepsilon = 23°.5$ in Istanbul, half-day remnant F, is calculated as $F = \sin^{-1}[(\tan \varphi).(\tan \varepsilon)] = 22°.2$. Accordingly, to calculate Y_i and O_i points, equation of daylight should be added to or subtracted from h_i angles. The expressions for calculation of these points are equal to the ones used in calculating I_i points.

2.1.1. Drawing Hour Lines Elapsed after sunrise [Dāʿir min al- shurūq] and Hour Lines remaining to sunset [Dāʿir ilā al- ghurūb]

For this, Y_i points of the beginning of Cancer, and O_i points of the beginning of Capricorn should be calculated. Yet, to find these, first, I_i equinox points, then I_{yi} Cancer and I_{oi} Capricorn zawālī shade points , and then I_{yyi} Cancer and I_{ooi} Capricorn ghurubi shade points need to be calculated.

Using the expressions derived above, let us give which relevant relations are used to calculate the values that give the important points of a horizontal sundial that is to be used in Istanbul:

Since for equatorial sundials, each hour corresponds to 15^0 degrees, after the sunrise, angles regarding each hour on the equinox line being as $i = 1,2,\ldots 6$

$$h_i = 90° - i.15°$$

should be taken. As Istanbul's latitude is $\varphi = 41°$, from the expression (1), horizontal hour angles can be found from

$$s_i = \tan^{-1}[0,656.(\tan h_i)]$$

and from the expression (2) shade lengths with reference to equinox points can be found from

$$l_i/p = (2.01966)/(\cos s_i)$$

When these values are calculated with reference to K point where the stick stands, with the help of (3) and (4), for t_i angle and m_i/p shade lengths

$$t_i = \tan^{-1}[(2.32335).(\tan s_i)]$$

$$m_i/p = (0.86929)/(\cos t_i)$$

is obtained. Since in the work of *Ruhāma*, t_i angles are taken from the west point with reference to south it is evaluated as $(90° - t_i)$. Table 1 shows t_i angles of I_i shade points on the equinox line and m_i/p distances.

Table 1

i	I_i	h_i	s_i	l_i/p	t_i	$(90° - t_i)$	m_i/p
1	I_1	75°	67°.7820	$5^P.8090$	80°.0289	9°.9711≡ 09° 58' 16''	$5^P.0204$≡ 5^P 01' 44''
2	I_2	60°	48°.6488	$3^P.0570$	69°.2528	20°.7472≡ 20° 44' 50''	$2^P.4539$≡ 2^P 27' 14''
3	I_3	45°	33°.2649	$2^P.4154$	56°.7304	33°.2696≡ 56° 43' 49''	$1^P.5846$≡ 1^P 35' 04''
4	I_4	30°	20°.7438	$2^P.1597$	41°.3462	48°.6548≡ 48° 39' 14''	$1^P.1579$≡ 1^P 09' 28''
5	I_5	15°	9°.9693	$2^P.0506$	22°.2144	67°.7956≡ 67° 47' 44''	$0^P.9890$≡ 0^P 59' 20''
6	I_6	0°	0°	$2^P.0197$	0°	90°	$0^P.8693$≡ 0^P 53' 57''

Since for I_7, …, I_{11} points, which are on the east side of the *ruhāma*, sundial drawing is symmetrical with reference to north-south line, t_i angles are taken with reference to west point.

To calculate Y_i and O_i points, which are time after sunrise and time to sunset, respectively, the equation of daylight with the degree of 22°.2 needs to be added to or subtracted from h_i angles. Other expressions can be used same as in the calculation of I_i points. Table 2 shows the comparison of calculated and given points.

Table 2

i	Y_i/O_i	h_i	$(90° - t_{yi})$	m_{yi}/p	$(90° - t_{oi})$	m_{oi}/p
1	Y_1/O_1	97°.2/52°.8	– 22°/– 22° 46'	$5^P.8090$	80°.0289	09^P 58' 16''
2	Y_2/O_2	82°.2/37°.8	– 13°/– 13° 32'	$3^P.0570$	69°.2528	20^P 44' 50''
3	Y_3/O_3	67°.2/22°.8	– 04°/– 05° 31'	$2^P.4154$	56°.7304	56^P 43' 49''
4	Y_4/O_4	52°.2/07°.8	6°/ 4°	$2^P.1597$	41°.3462	48^P 39' 14''
5	Y_5/O_5	37°.2/0°	20°/ 15°	$2^P.0506$	22°.2144	67^P 47' 44''
6	Y_6/O_6	22°.2/0°	36°/ 25° 12'	$2^P.0197$	0°	90^P
7	Y_7/O_7	7°.2/0°	0°			
8	Y_8/O_8	0°	0°			
9	Y_9/O_9	0°	0°			
10	Y_{10}/O_{10}	0°	0°			

2.1.2. Drawing the Tropics of Cancer and Libra

The angle and lengths of the shade falling upon the equinox line for Ariel and Libra can be obtained by the line connecting $I_1 = I_{11}$ points. The calculated angle $(90° - t_1) = (90° - t_{11})$ = 09° 58' and shade length $m_1 = m_{11}$ = 12.(5^P 01' 44'') = 60^P 20 of these two points are equal to the one given in the text. (See *Table 1*)

2.1.3 Drawing Hour lines to Midday and Hours after Midday [Fażl al- dā'ir]

To draw the hour lines for hours to and after midday, beginning points of I_{yi} Sagittarius and I_{oi} Capricorn which corresponds to I_i equinox points that are formulated with reference to K points given above need to be calculated. From the expressions given above, these relations can be calculated as follows:

$w_i = \sin^{-1}[(1{,}5243)/(l_i/p)]$

$l_y/p = (1{,}3978)/[\sin(w + 23{,}5°)]$

$l_o/p = (1{,}3978)/[\sin(w - 23{,}5°)]$

$x = \sin^{-1}\{[(1{,}1504).(\sin s_i)]/(m/\mathrm{p})\}$

$t_{yi} = \tan^{-1}\{[\sin(x_i - t_i)]/\{[1/[(0{,}869)\ (l_{yi}/p)]] - [\cos(x_i - t_i)]\}\}$

$t_{oi} = \tan^{-1}\{[\sin(x_i - t_i)]/\{[1/[(0{,}869)/(l_{oi}/p)]] - [\cos(x_i - t_i)]\}\}$

$m_{yi}/p = [(l_{yi}/p).(\sin s_i)]/(\sin t_{yi})$

$m_{oi}/p = [(l_{oi}/p).(\sin s_i)]/(\sin t_{oi})$.

The values regarding the shade angles (90° - $t_{y(6\text{-}i)}$) and (90° - $t_{o(6\text{-}i)}$) of I_{yi} and I_{oi} points that are evaluated, and the shade lengths of $m_{y(6\text{-}i)}$ and $m_{o(6\text{-}i)}$ are compared to those given in the text in *Table 3*:

Table 3

Hour	**Hours to Midday**	**Hours after Midday**	**(90° – $t_{y(6\text{-}i)}$) / (90° – $t_{o(6\text{-}i)}$)**		**$m_{y(6\text{-}i)}$ / $m_{o(6\text{-}i)}$**	
			Calculated	**Given**	**Calculated**	**Given**
6	I_{yo}-I_{yo1}	I_{y12}-I_{yo11}	– 18° 10' / – 13°	– 18° 10' / – 13°	44^P 17' / 62^P	44^P 17' / 62^P
5	I_{y1}-I_1	I_{y11}-I_{11}	– 08°.19' / 09° 58'	**– 12° / -**	24^P / 29^P 27'	**25^P 15' / -**
4	I_{y2}-(I_2)-I_{o2}	I_{y10}-(I_{10})-I_{o10}	– 00° 00' / 37° 09'	**8° 22' /-**	15^P 41' /	**14^P / -**
3	I_{y3}-(I_3)-I_{o3}	I_{y9}-(I_9)-I_{o9}	10° 52' / 48° 15'	10° /	10^P 33' /	-
2	I_{y4}-(I_4)-I_{o4}	I_{y8}-(I_8)-I_{o8}	25° 65' / 60° 51'	25° /	7^P 05' /	6^P / -
1	I_{y5}-(I_5)-I_{o5}	I_{y7}-(I_7)-I_{o7}	49° 44' / 74° 56'	49° 44' /	4^P 40' /	**58^P 45' / -**
0	I_{y6}-(I_6)-I_{o6}		90°	-	3^P 47' /	-

2.1.4 Drawing the Curves of Asr-i Awwal and Asr-i Thānī

In *ruhame*, there are two curves for *asr* prayer time defined as *asr-i awwal* and *asr-i thānī*. If the length of sundial stick p and the highest point of the Sun $m_6 = (a\text{-}b)$ are given, the shade length for *asr-i awwal* in that day can be calculated by $a_1 = (b\text{-}d) = (m_6 + p)$, and that of *asr-i thānī* can be calculated by $a_2 = (b\text{-}e) = (m_6 + 2.p)$ (Figure 6). If the distances are proportioned to the stick length p, as is the case in *ruhāma*, the following expressions can be obtained:

$a_1/p = (m_6/p + 1)$

$a_2/p = (m_6/p + 2)$.

Since the midday shades defined with reference to the point K are already calculated for equinox and tropics, it is easy to find a_1/p and a_2/p distances. Yet, to find the t_1 and t_2 angles, the expressions given above should be calculated in accordance to these distances.

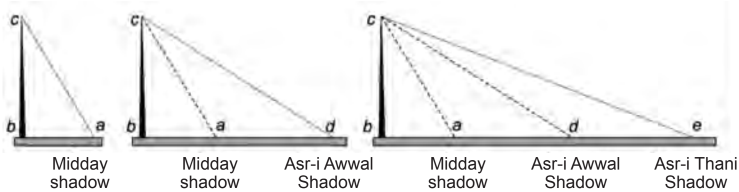


Figure 6 Definitions of the Time of Asr-i Awwal and Asr-i Thani

Table 4 shows the calculated and shade lengths given in the text, for the points of *asr-i awwal* and *asr-i thāni*. Some of the values given in the original text, which aremarked with red in the table are incorrect. They could be the result of incorrect copying (*istinsākh*).

For the A_i points on the equinox line, from the expression (4)

$t_i = \cos^{-1}[(\tan \varphi)/(a_i/p)]$,

The easiest way to determine A_{iy} and A_{io} points that are at the beginning of Cancer and Capricorn is determining taiy and taiy angles on the drawing of *ruhame* by drawing circles with K as the center and aiy and aio as radiuses, and by guessing their intersection points with the hyperbolic curve. Otherwise, hyperbolic equations regarding Cancer and Capricorn need to be found and intersected with the aforesaid circle. *Table 5* summarizes the results regarding t_{aiy} and t_{aio} angles.

Table 4

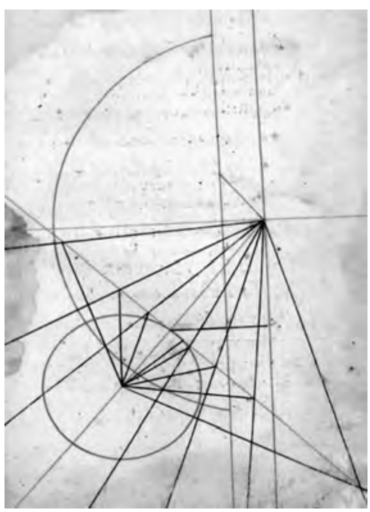
Point A_i	**Definition** $\boldsymbol{a_i}$	**Calculated**		**Given**
		$\boldsymbol{a_i/p}$	$\boldsymbol{a_i}$	$\boldsymbol{a_i}$
A_{1y}	$a_{1y}/p = m_{y6}/p + 1$	$1^P.32$	$15^P\ 47'$	$15^P\ 47'$
A_1	$a_1/p = m_6/p + 1$	$1^P.87$	$22^P\ 26'$	$22^P\ 26'$
A_{1o}	$a_{1o}/p = m_{o6}/p + 1$	$3^P.10$	$37^P\ 35'$	$36^P\ 48'$
A_{2y}	$a_{2y}/p = m_{y6}/p + 2$	$2^P.32$	$27^P\ 47'$	$26^P\ 48'$
A_2	$a_2/p = m_6/p + 2$	$2^P.87$	$34^P\ 26'$	$34^P\ 26'$
A_{2o}	$a_{2o}/p = m_{o6}/p + 2$	$4^P.10$	$49^P\ 10'$	$49^P\ 10'$

Table 5

Point A_i	**Expression** $\boldsymbol{t_{ai}}$	**Calcuated or Measured** $\mathbf{90° - t_{ai}}$	**Given** $\mathbf{90° - t_{ai}}$
A_{1y}	-	0° 10'	0° 10'
A_1	$t_{a1} = \cos^{-1}[(\tan \varphi)/(a_1/p)]$	27°.70 ≡ 27° 42'	27° 48'
A_{1o}	-	57°	56° 48'
A_{2y}	-	-11°	–11° 32'
A_2	$t_{a2} = \cos^{-1}[(\tan \varphi)/(a_2/p)]$	17°.63 ≡ 17° 38'	09° 26'
A_{2o}	-	49°	49° 10'

Conclusion

Based upon the results obtained above, if a table of angles and distances of hour lines is prepared without any mistake, it is very easy to practically draw a horizontal sundial or *ruhāma* for Istanbul. Since in the drawing the author added to the end of the text, that the north-south line or midday line is drawn with a slope of 45 degrees to the right side of the paper, it gives a false impression to the reader that it is a vertical sundial (*Picture 3*). Even in vertical sundials east-west line is drawn parallel to the lower edge of paper. On the other hand, in horizontal sundials, equinox line is usually drawn parallel to the lower edge of paper. Given that, it is still a mystery how the author calculated these values and wrote the text.



Picture 3 Sundial drawing at the end of the text.

11 In the original text, 10° is written instead of 10' by mistake.

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Appendix

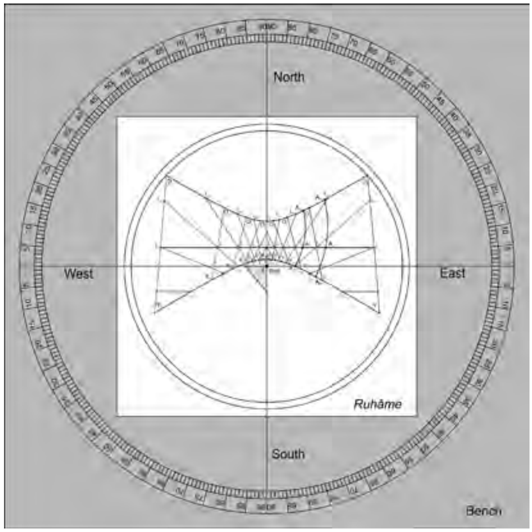


Figure A: Board used in drawing ruhāma

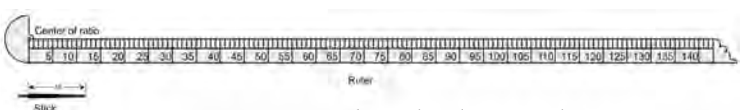


Figure B: Drawing scale used in drawing ruhāma

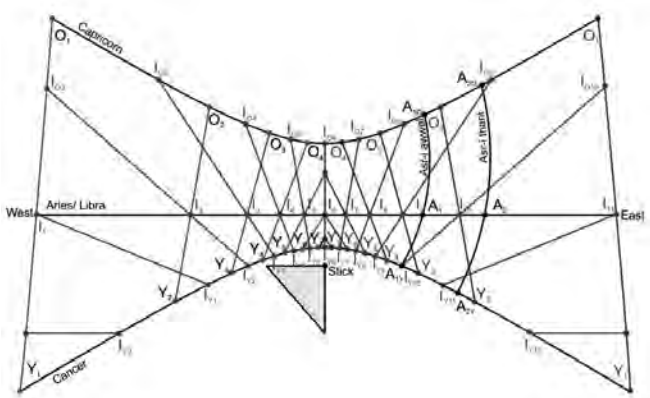
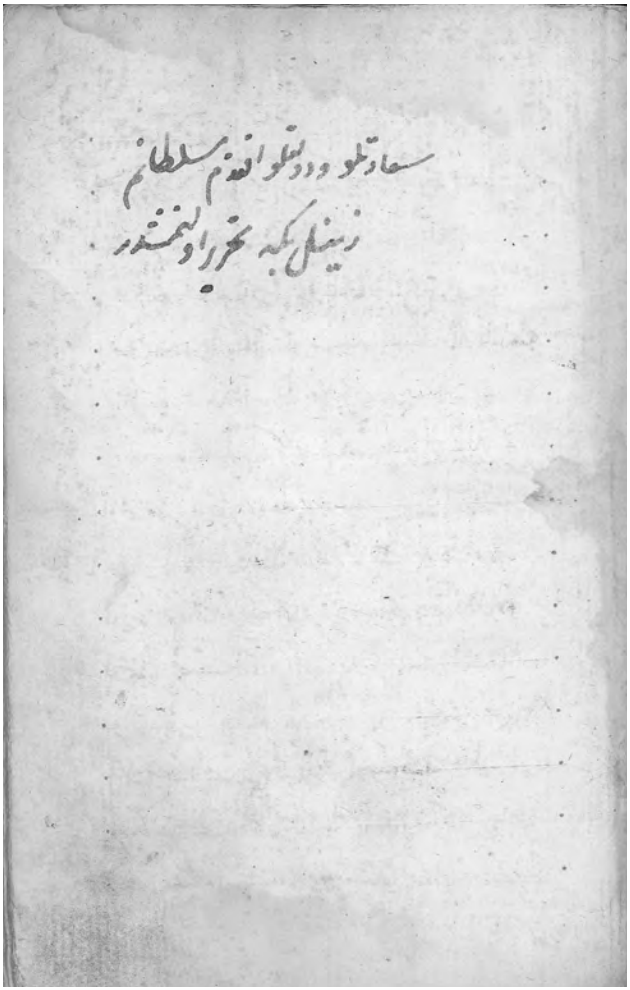


Figure C: Points determined in drawing sundial on ruhāma

Facsimile of Ruhāma



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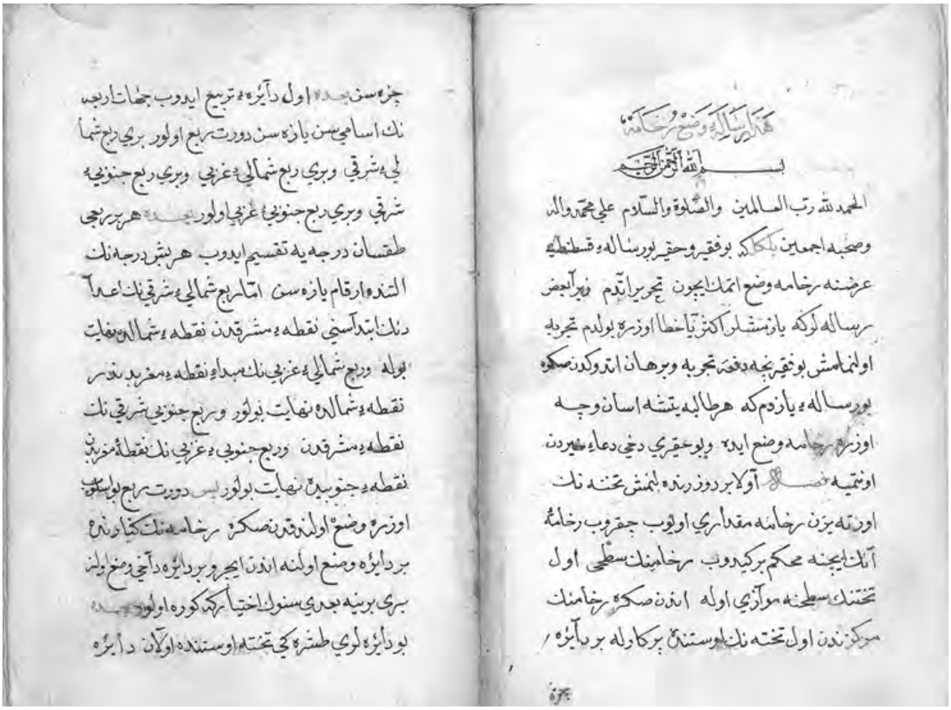

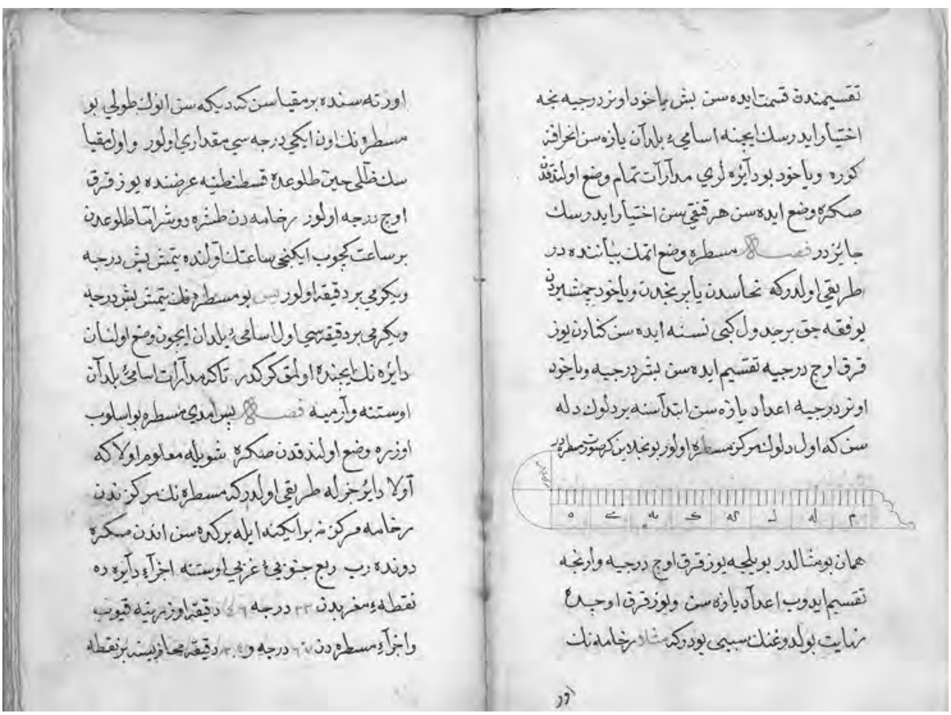

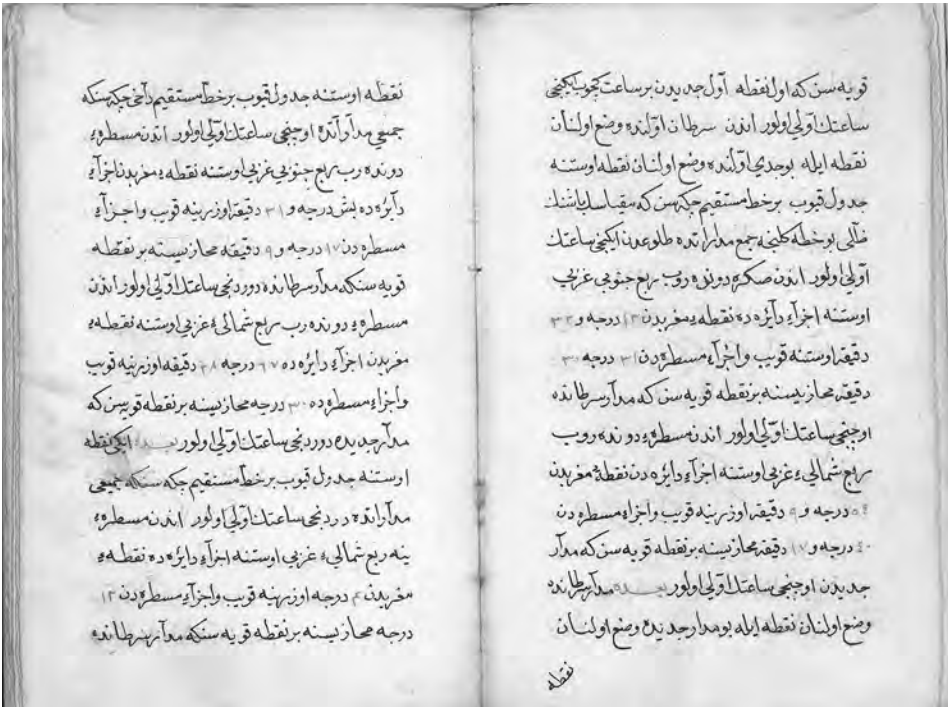

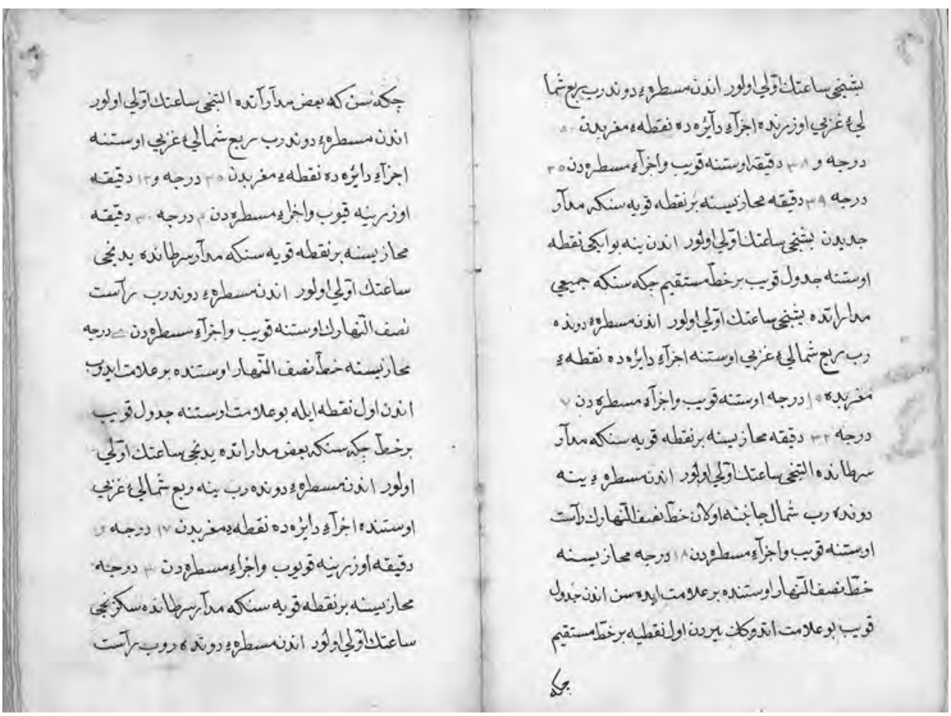

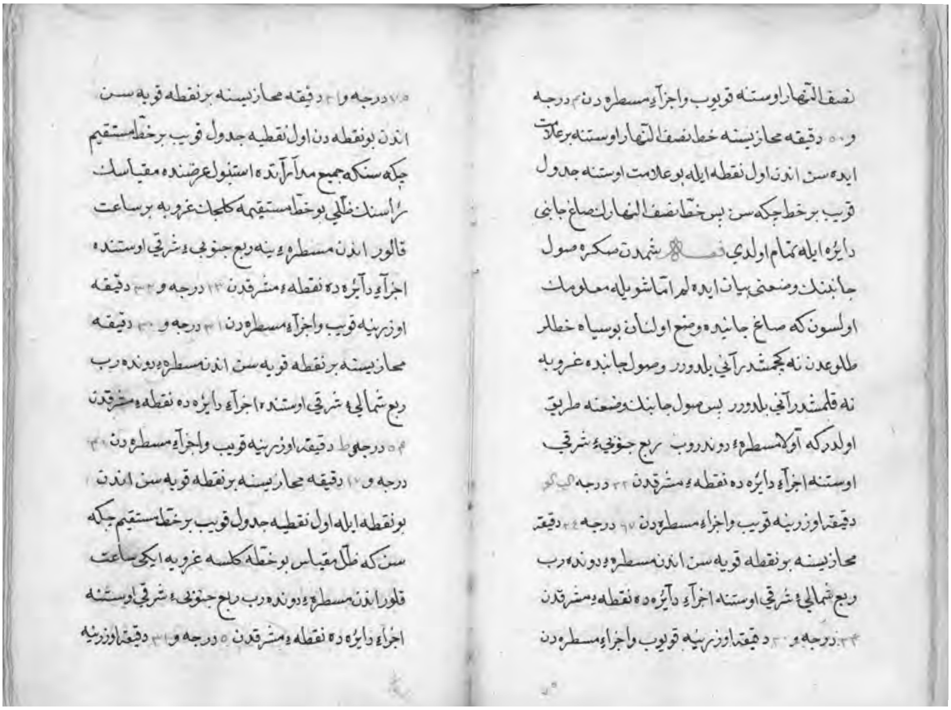
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و ٥٠ دقيقه محاذيسنه خط نصف النهار اوستنه بر علامت
ايده سن اندن اول نقطه ايله بو علامت اوستنه جدول
قويوب بر خط چكه سن پس خط نصف النهارك صاغ جانبى
دايره ايله تمام اولدى … شمدنصكره وصول
جانبنك وضعنى بيان ايده لم اما شونيله معلوم
اولسون كه صاغ جانبده وضع اولنان بو سياه خطلر
طلوعدن نه قدر … آنى بيلدورر وصول جانبده غروبه
نه قلمشدر آنى بيلدورر پس وصول جانبنك وضعنه طريق
اولدركه اولا مسطره يى دوندره روب ربع جنوبى شرقى
اوستنه اجزاء دايره ده نقطه مشرقدن ٢٢ درجه …
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اندن بو نقطه دن اول نقطه يه جدول قويب بر خط مستقيم
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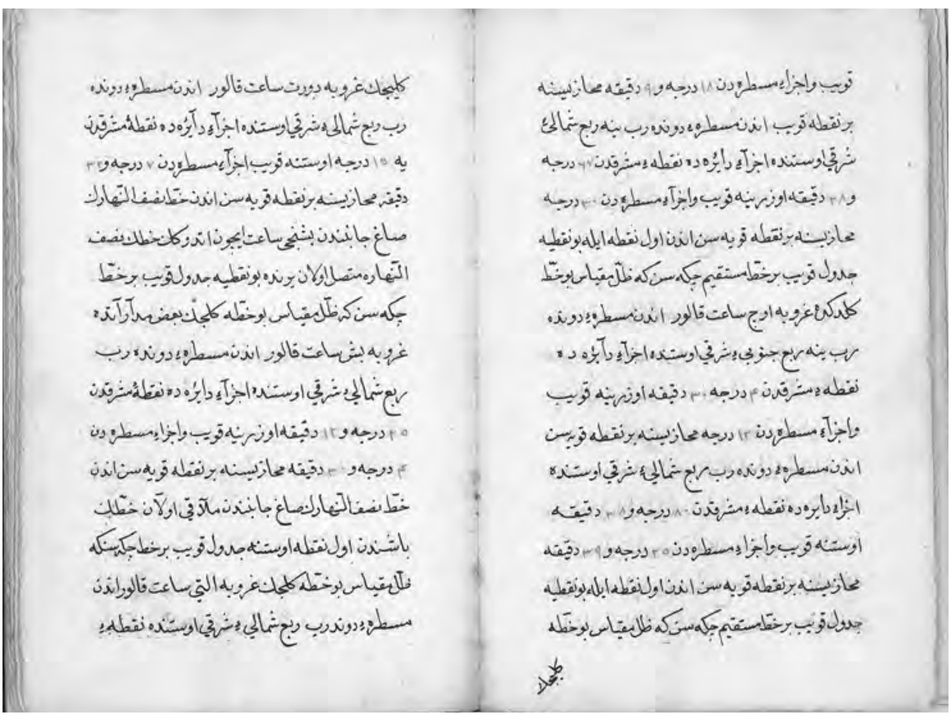
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كلجك

كلجك غروبه دورت ساعت قالور اندن مسطره يى دوندره
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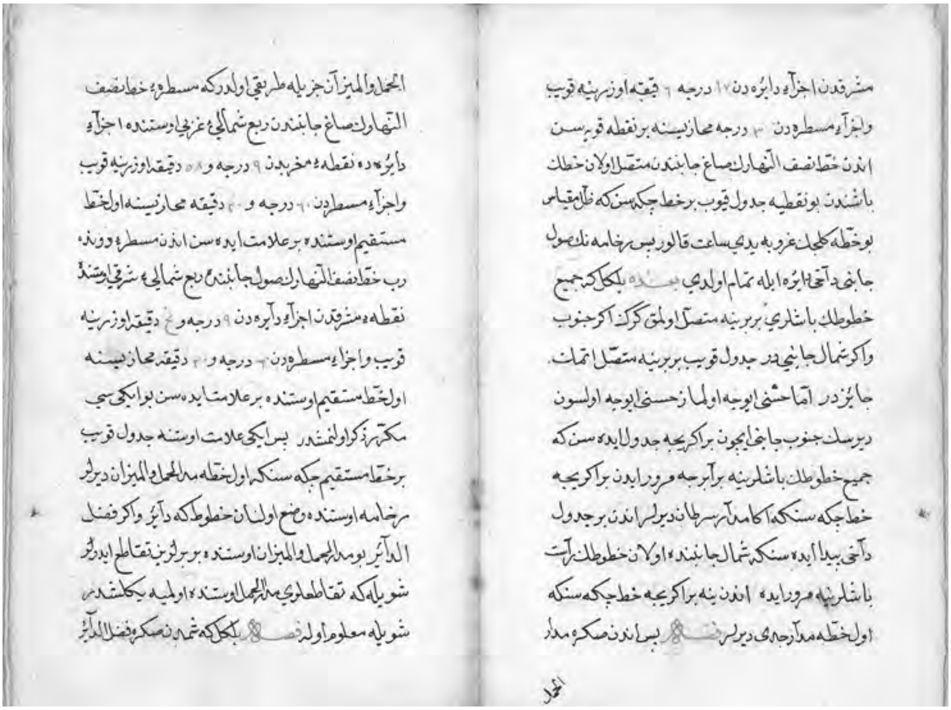

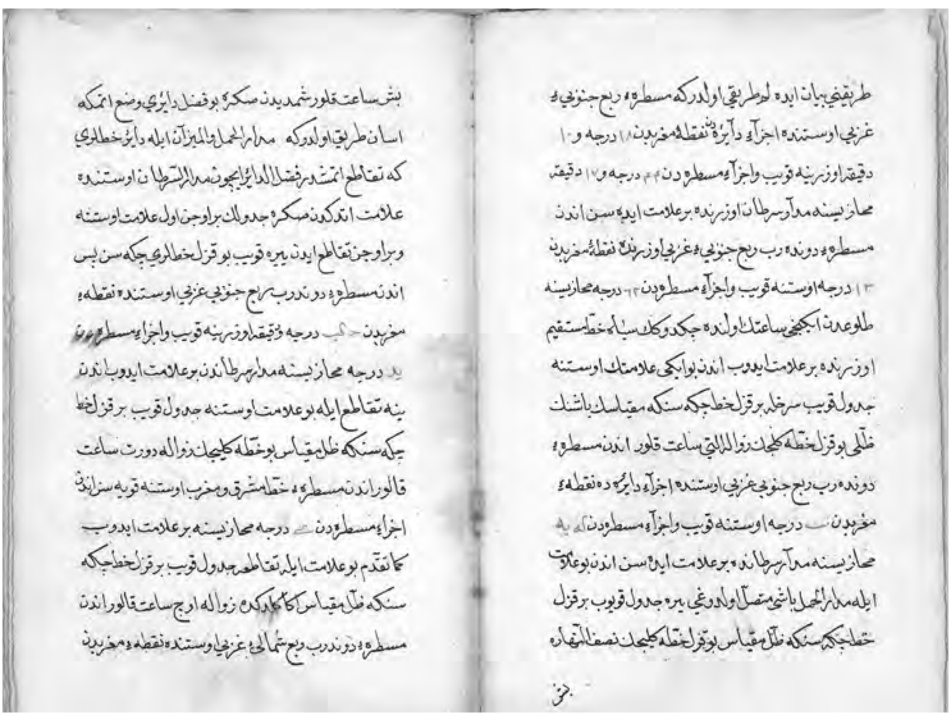

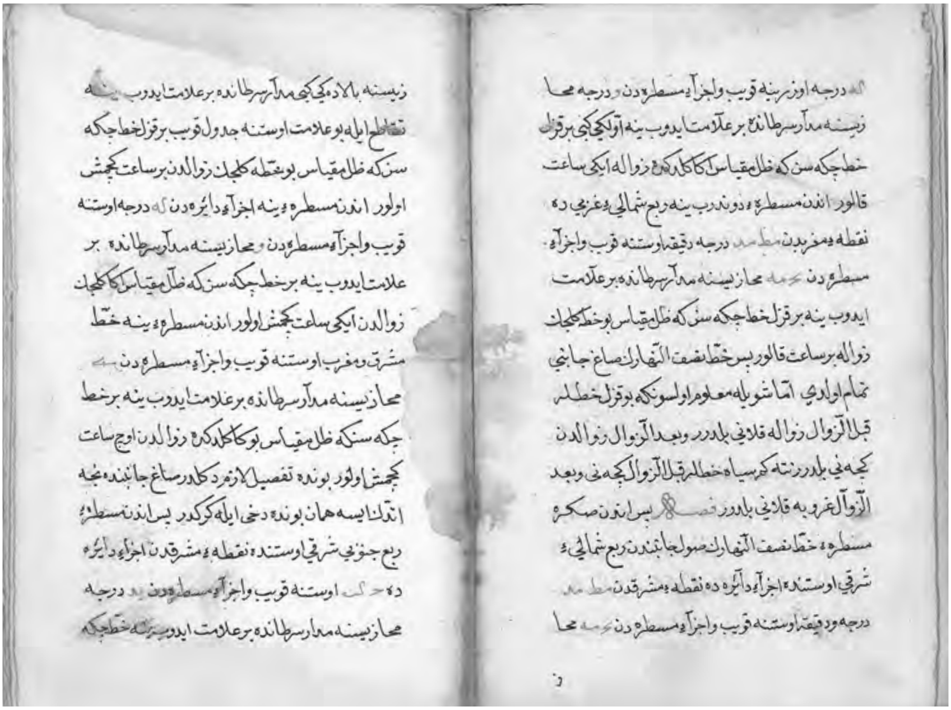

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خط چکه سن که ظل مقیاس کاکلدکده زواله ایکی ساعت
قالور اندن مسطره‌ی دونده رب ینه ربع شمالی غربی ده
نقطه مغربدن [illegible] درجه دقیقه اوستنه قویب واجزاء
مسطره دن [illegible] محاذیسنه مدار سرطانده بر علامت
ایدوب ینه بر قزل خط چکه سن که ظل مقیاس بو خطه کلدیکی
زواله بر ساعت قالور بس خط نصف النهارك صاغ جانبی
تمام اولدی اما شونی معلوم اولسونکه بو قزل خطلر
قبل الزوال زواله قلانی بلدرر وبعد الزوال زوالدن
کچه نی بلدررزنته کم سیاه خطلر قبل الزوال کچه نی وبعد
الزوال غروبه قلانی بلدورر فصل بس اندن صکره
مسطره‌ی خط نصف النهارك صول جانبندن ربع شمالی
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درجه ودقیقه اوستنه قویب واجزاء مسطره دن [illegible] محا

ذیسنه بالاده کی کبی مدار سرطانده بر علامت ایدوب ینه
تقاطع ایله بو علامت اوستنه جدول قویب بر قزل خط چکه
سن که ظل مقیاس بو نقطه کلدیکی زوالدن بر ساعت کچمش
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کچمش اولور بونده تفصیل لازم دکلدر صاغ جانبده نیجه
اتدك ایسه همان بونده دخی ایله کرکدر بس اندن مسطره‌ی
ربع جنوبی شرقی اوستنده نقطه مشرقدن اجزاء دایره
ده [illegible] اوستنه قویب واجزاء مسطره دن [illegible] درجه
محاذیسنه مدار سرطانده بر علامت ایدوب ینه خط چکه

که ظل مقیاس بو کاکلدکده زوالدن دورت ساعت کچمش
اولور اندن مسطره‌ی اجزاء دایره دن [illegible] اوستنه قویب
واجزاء مسطره دن [illegible] محاذیسنه مدار سرطانده بر
علامت ایدوب اندن بو علامت ایله مدار الحمل والمیزان
باشنه جدول قویب بر خط چکه سنکه ظل مقیاس
بو کاکلدیکی زوالدن بش ساعت کچمش اولور اندن مسطره‌ی
اجزاء دایره دن نقطه مشرقدن [illegible] درجه اوستنه قویب
اجزاء مسطره دن [illegible] درجه محاذیسنه غروبه بر ساعت
قلان خط مستقیم اوستنده بر علامت ایده سن اندن
مسطره‌ی دونده رب اجزاء دایره دن نقطه مشرقدن
[illegible] اوستنه قویب واجزاء مسطره دن [illegible] محا
ذیسنه مدار سرطانده بر علامت ایدوب اندن بو ایکی
علامت اوستنه جدول قویب بر قزل خط چکه سنکه
ظل مقیاس بو خطه کلدیکی زوالدن التی ساعت کچمش اولور
بس برخامه نك دایره‌سی وفضل الدایره‌سی تمام اولدی اندن
صکره

صکره دایره وفضل الدایر خطلرینك اوستنه رقملرین
یازه سن فصل بیلکل که برخامه ایکی قوس وضع ایده
سن برینه عصر اول وبرینه عصر ثانی درلر مقیاسك
ظلی قوس عصر اوله کلدیکی امامین قتنده عصر اولك وقتی
اولور وقوس عصر ثانیه کلدیکی امام اعظم قتنده عصر ثانینك
وقتی اولور شمدی امامین مذهبی ایله عمل ایدرلر بس
امدی قوس عصر اوله طریقی اولدر که مسطره‌ی دونده رب
ربع جنوبی شرقی اوستنده اجزاء دایره ده نقطه مشرقدن
[illegible] درجه اوستنه قویب واجزاء مسطره دن [illegible] درجه
ودقیقه محاذیسنه مدار سرطانده بر علامت ایده سن اندن
مسطره‌ی دونده رب ربع شمالی شرقی اوستنده اجزاء دایره ده
نقطه مشرقدن [illegible] درجه ودقیقه اوزرینه قویب واجزاء
مسطره دن [illegible] محاذیسنه مدار الحمل والمیزان اوستنده
بر علامت ایده سن اندن مسطره‌ی دونده رب ربع شمالی شرقی
اوستنده اجزاء دایره ده نقطه مشرقدن [illegible] اوستنه قویب