NAZARİYAT

Preparation for Astronomy: ʿAlī Qushjī's Risāla dar ʿIlm-i Ḥisāb

Zehra Bilgin*

Abstract: Alī Qūshjī (d. 879/1474), a fifteenth-century astronomer, mathematician, theologian, and linguist, played a key role in the formation of Ottoman scientific life. This role has been analyzed in the history of science and in the history of thought more broadly. Upon Sultan Mehmed II's invitation, he contributed to the formation of the Fatih Semâniye and Ayasofya Madrasas by transferring the scholarly accumulation of the Samarkand mathematics-astronomy school to Istanbul, and his works were widely used in Ottoman intellectual circles for centuries.

Risāla dar ʿIlm-i Hisāb, which is the first of two mathematical works by Alī Qūshjī, was written in Persian, probably in Samarkand, sometime before 861/1456. This work has a remarkable place in the Islamic mathematical tradition given its numerous copies, its widespread distribution in Iran and Anatolia, and its being used as a textbook for centuries. In this work, an analysis based on several copies of the work will be shared, correcting outdated information in the literature and clarifying the place and value of *Risāla dar ʿIlm-i Hisāb* in terms of the history of science. As an example of the work's content, the method of finding the quadratic root of integers will be examined and the proof of the method will be given.

Keywords: Alī Qūshjī, history of mathematics, Ottomans, arithmetic, Risāla dar ʿIlm-i Hisāb

* Assist. Prof., Istanbul Medeniyet University, Institute for the History of Science. Correspondence: zehrabilgin.zb@gmail.com

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1. Introduction and Background

There are only two kinds of stories: A man goes on a journey, or a stranger comes to town.

This article revolves around a segment of the story of a man from Samarkand who arrived in Constantinople in 1473—a narrative that has maintained its distinguished place in the historiography of Ottoman science up to the present day. When Alī Qūshjī was sent to Istanbul as an envoy of Uzun Hasan, the ruler of the Aq Qoyunlu dynasty, it is unknown whether he anticipated that, just as he had once been under the patronage of Ulugh Beg, the Timurid prince, he would now find himself under the protection of another scholar-ruler, Sultan Mehmed II. It remains uncertain whether he foresaw that he would spend the rest of his life in Istanbul, which was emerging as a burgeoning center of knowledge, or that his works and teachings would continue to influence the world of science for centuries to come.

On the other hand, historical records confirm that his arrival was eagerly anticipated in Istanbul, suggesting that Sultan Mehmed the Conqueror's foresight in this matter was remarkably astute. Even before his arrival, Alī Qūshjī had been renowned as a scholar whose fame had spread through his works. However, by accepting Sultan Mehmed II's invitation and positioning himself at the heart of Istanbul's newly developing intellectual scene, he ensured that his reputation would extend far beyond his lifetime. He would become a pivotal reference point in the Ottoman Empire for both mathematical sciences and religious studies for centuries.

The effort and success of Amir Timur (r. 1370–1405, d. 807/1405) in transforming Samarkand into a center of knowledge during his reign are well known. After his grandson Ulugh Beg (d. 853/1449) assumed power, he consolidated Samarkand's preeminent role in the intellectual world by founding the Samarkand Madrasa and the Samarkand Observatory. This transformation established a new school of mathematics and astronomy in Islamic mathematical history, turning Samarkand into an intellectual hub for mathematical sciences of its era.[1] Ulugh Beg's authority as a ruler, combined with

1 For an examination of the school of mathematics and astronomy, see İhsan Fazlıoğlu, "The Samarqand Mathematical and Astronomical School A Basis for Ottoman Philosophy and Science," *Journal for the History of Arabic Science* 14 (2008): 3–68.

his scholarly identity and passion for knowledge, created an environment of patronage that attracted and satisfied the scholars of the time. Distinguished figures from various regions of the Islamic world thus naturally gathered in Samarkand.

Among the notable scholars associated with mathematical sciences at the Samarkand Madrasa and its surrounding intellectual circle were Qadi-zāde al-Rūmī (d. after 844/1440), Jamshīd al-Kāshī (d. 832/1429), Fathullah al-Shirvanī (d. 891/1486), and Ulugh Beg himself. Joining them first as a student and later as a colleague was Alī Qūshjī (d. 878/1474), the protagonist of this narrative. These scholars collaborated on a canonical work, the *Zīj of Ulugh Beg*, whose astronomical observations and calculations would resonate for centuries. When the project was left incomplete due to the passing of the first two scholars, Alī Qūshjī continued the endeavor alongside Ulugh Beg, contributing significantly to its completion.

In this prominent center of mathematical sciences within the region of Transoxiana, several of the most important works of Islamic mathematical literature were authored. In addition to the *Zīj of Ulugh Beg*, notable examples include Qadi-zāde al-Rūmī's commentaries on Shams al-Din al-Samarqandī's *Ashkāl al-Tȧsīs* and Mahmud ibn Umar al-Chaghmīnī's *al-Mulakhkhas fi'l-Hay'a*, as well as Jamshīd al-Kāshī's remarkable mathematical treatise, *Miftāh al-Hisāb*. For the aforementioned scholars, these works are considered their *magna opera*.

This school of thought placed astronomy at its core and regarded mathematics primarily as an introduction to astronomy. Like his teachers and colleagues, Alī Qūshjī contributed to this tradition with his works in astronomy and mathematics. During his time in Samarkand, he wrote *Risāla dar 'Ilm-i Hay'a* on astronomy and *Risāla dar 'Ilm-i Hisāb* on arithmetic, both in Persian. After his migration to Istanbul, he composed two more works on the same subjects in Arabic: *al-Risāla al-Fathiyya fī 'Ilm al-Hay'a* and *al-Risāla al-Muhammadiyya fī 'Ilm al-Hisāb*.[2]

In 2019, *al-Risāla al-Fathiyya* was the focus of Hasan Umut's doctoral dissertation at McGill University.[3] Although *Risāla dar 'Ilm-i Hay'a* was not the main subject of this thesis, Umut's comparisons with *al-Risāla al-Fathiyya* provide some insights into

2 For autoghraph copies, see Alī Qūshjī, *al-Risāla al-Fathiyya fī 'Ilm al-Hay'a,* Süleymaniye Manuscript Library, Ayasofya 2733/1, f. 1b-70a; Alī Qūshjī, *al-Risāla al-Muhammadiyya fī 'Ilm al-Hisāb,* Süleymaniye Manuscript Library, Ayasofya 2733/2, f. 73b-168b.

3 Umut, "Theoretical Astronomy in the Early Modern Ottoman Empire ʿAlī al-Qūshjī's *Al-Risāla al-Fatḥiyya*." (PhD diss., McGill University, 2019).

the former's content. In addition to these two texts, to Alī Qūshjī are also attributed several smaller treatises on specific geometric issues in the history of Ottoman mathematical literature. These include *Risāla fī Istikhrāji Maqādir al-Zāwiya min Maqādir al-Azlā*, which explains methods for calculating the angles of triangles with known side lengths; *Risāla fī Qawā'id al-Hisābiyya wa'l-Dalā'il al-Handasiyya*, addressing certain rules and theorems in arithmetic and geometry; and *Risāla fi'l-Zāwiya al-Hādda idha Furiḍat Harakatuhu Ahadi Dil'ayha Tahsulu Zāwiya Munfarija*, which discusses a problem debated in the Sultan's court about whether an acute angle becomes obtuse when one of its sides is moved.[4] Ihsan Fazlıoğlu has demonstrated that this last treatise should be attributed to Sinan Pasha.[5]

Amidst this scholarship, *al-Risāla al-Muhammadiyya* has not been fully studied or published. Partial studies include Elif Baga's 2012 doctoral dissertation at Marmara University, in which she provided a brief content summary and analyzed the algebra section of the work.[6] Additionally, Ihsan Fazlıoğlu examined relevant sections in his article, "The 'Double False Position' and 'Analysis' Methods in Alī Qūshjī's *al-Muhammadiyya fi'l-Hisāb*."[7]

Risāla dar 'Ilm-i Hisāb appears to have been overshadowed in contemporary literature by *al-Risāla al-Muhammadiyya*. Adnan Adıvar claims that the latter is a direct translation of the former.[8] Similarly, Salih Zeki, in *Kāmūs-i Riyāziyyāt*, states that Alī Qūshjī "translated it [*Risāla dar 'Ilm-i Hisāb*] into Arabic from a previously written Persian version when he arrived in Istanbul."[9] Cevat İzgi describes *al-Risāla al-Muhammadiyya* as an expanded translation.[10] Ihsan Fazlıoğlu adopts a more cautious stance in the *İslam Ansiklopedisi*, stating only that *Risāla dar 'Ilm-i Hisāb* forms the basis of *al-Risāla al-Muhammadiyya*.[11]

4 Ekmeleddin İhsanoğlu et al., *Osmanlı Matematik Literatürü Tarihi* (İstanbul: IRCICA Yayınları, 1999), 1:24–6.

5 İhsan Fazlıoğlu, "Ali Kuşçu'nun bir hendese problemi ve Sinan Paşa'ya nisbet edilen cevabı," *Divan: Disiplinlerarası Çalışmalar Dergisi* 1 (December 1996): 85–106.

6 Elif Baga, "Osmanlı Klasik Dönemde Cebir" (PhD diss., Marmara University, 2012), 78–82.

7 İhsan Fazlıoğlu, "Ali Kuşçu'nun el-Muhammediyye fî el-hisâb'ının "Çift yanlış" ile "Tahlîl" hesâbı bölümü," *Kutadgubilig* 4 (October 2003): 135–55.

8 Adnan Adıvar, *Osmanlı Türklerinde İlim* Edition (İstanbul: Remzi Kitabevi, 1982), 4:49.

9 Salih Zeki, *Kâmûs-i Riyâziyyât*, İstanbul Üniversitesi Library Rare Works Collection TY00915, 8:1030.

10 Cevat İzgi, *Osmanlı Medreselerinde İlim* (İstanbul: Küre Yayınları, 2019), 207.

11 İhsan Fazlıoğlu, "er-Risâletü'l-Muhammediyye," in *TDV İslam Ansiklopedisi*, accessed December 31, 2023, https://islamansiklopedisi.org.tr/er-risaletul-muhammediyye.

The manuscript of *Risāla dar ʿIlm-i Hisāb*, based on the autograph copy, along with its Turkish translation and mathematical analysis, was made available to readers by Zehra Bilgin in her 2024 doctoral dissertation at Istanbul Medeniyet University.[12] *Al-Risāla al-Muhammadiyya* has neither been published nor fully translated into any modern language.

This article focuses on the relationship and differences between *Risāla dar ʿIlm-i Hisāb* and *al-Risāla al-Muhammadiyya*. It is hoped that such a comparison will test the validity of previous claims in the literature and help correct any inaccuracies. Beyond shedding light on whether *al-Risāla al-Muhammadiyya* is merely a translation or an expanded version of *Risāla dar ʿIlm-i Hisāb*, this analysis will also reveal the evolution of Alī Qūshjī's pedagogical approach to arithmetic. Considering that approximately twenty years likely separate the composition of the two works, the study will also reflect changes in the needs of the intellectual circles in which he moved over time.

Risāla dar ʿIlm-i Hisāb is the first work in which Alī Qūshjī's understanding of mathematics is fully articulated. He wrote it in Persian during the second half of the fifteenth century, probably in Samarkand. There are nearly four hundred known copies of this manuscript in libraries worldwide, most of which are housed in Türkiye and Iran.[13] The large number of surviving copies, the continuation of manuscript copying until the nineteenth century, and its printed edition under the title *Mizān al-Hisāb*[14] in Iran in 1850 attest to the widespread readership and significance of *Risāla dar ʿIlm-i Hisāb* within the Islamic mathematical tradition.

The information available in the current literature on *Risāla dar ʿIlm-i Hisāb* is insufficient to fully determine its historical scholarly significance and importance. No critical edition or translation of the work has yet been published.[15] Additional-

12 Zehra Bilgin, "Hesab Bilimine Giriş: Ali Kuşçu'nun *Risâle der İlm-i Hisâb* Adlı Eseri- Tenkitli Metin, Çeviri, Değerlendirme-." (PhD diss., İstanbul Medeniyet Üniversitesi, 2024).

13 Mustafa Derayâti, *Fankha* (*Fihrist Nusakh Khatti Iran, Iranian Manuscripts Index*) (Tehran: National Library of Iran, 1391), 12:962–80; İhsanoğlu et al., *Osmanlı Matematik Literatürü Tarihi,* 21–4.

14 İhsanoğlu et al., *Osmanlı Matematik Literatürü Tarihi,* 21.

15 In the bibliographical work *Mathematicians, Astronomers, and Other Scholars of Islamic Civilization and Their Works* (*7th-19th c.*) by Boris Rosenfeld and Ekmeleddin İhsanoğlu, the entry for "Alī Qūshjī" mentions that his *Risāla dar ʿIlm-i Hisāb* was translated into Russian by Atayeva. A reference is made to the bibliography section about Qūshjī. In this reference, Alī Qūshjī's astronomy treatise is found in its Uzbek translation, *Astronomiya oid risola,* by I. M. Muminov in 1968: *Ali Kuşçu, Astronomiya oid risola,* trans. I. M. Muminov (Tashkent: Uzbekiston Ssr "Fan" Naşrieti, 1968). On

ly, questions persist about the writing process used for the work. This study utilizes twenty-six manuscripts collected from various locations worldwide, predominantly from Türkiye. One key research question is whether versions other than the author's autograph manuscript—housed at the Süleymaniye Library under Ayasofya 2733/3—exist.[16] This article seeks to answer this question by analyzing the accessible manuscripts and sharing findings on the historical and scholarly context of the work.

The term "science of arithmetic" (*ʿIlm-i Hisāb*) in the work's title refers to one of the main branches of classical mathematics. This branch studies the relationships between numbers and covers operations such as addition, subtraction, multiplication, division, powers, and roots. *Ilm-i hisāb*, inherited from Indian and Greek traditions, reached its near-natural limits by the fifteenth century owing to the contributions of Islamic mathematicians. In addition to the Indian arithmetic system, introduced to the Islamic world by Muhammad ibn Mūsā al-Khwarizmī (d. after 847) and later to Latin Europe, other methods such as mental arithmetic (*hisāb al-hawāʾi*) and sexagesimal arithmetic (*hisāb sittini*, also known as *hisāb al-tanjīm* or *hisāb al-munajjimīn*) were prevalent.[17]

In the Islamic mathematical tradition, some works focus exclusively on one type of arithmetic, while others encompass several methods. Alongside the arithmetic methods mentioned, these works frequently include sections on mensuration (*misāha*). This branch, which can be classified as mensuration, involves measuring lengths, areas, and volumes of objects. Although the connection between mensuration and arithmetic may not be immediately apparent, calculations of numerical values for lengths, areas, and volumes using various units of measurement are fundamentally arithmetic operations. Therefore, mensuration can be considered a subfield of calculation.

the other hand, Rosenfeld and İhsanoğlu's work also notes that Atayev translated *al-Risāla al-Muhammadiyya* into Russian. This reference includes U. Atayeva's Russian work titled *Arithmeticheskiy traktat*. Boris Rosenfeld and Ekmeleddin İhsanoğlu, *Mathematicians, Astronomers, and Other Scholars of Islamic Civilization and Their Works (7th-19th c.)* (Istanbul: IRCICA, 2003), 286, 665. Since Atayeva's work has not yet been accessed, it has not been determined whether the translation pertains to *al-Risāla al-Muhammadiyya* or *Risāla dar Ilm-i Hisāb*. In the entry on Alī Qūshjī in *Matematicheskiye i astronomicheskiye rukopisi uchonykh Sredney Azii X-XVIII* by G. P. Matviyevskaya and H. Tllashev, it is mentioned that the content of *Risāla fī ʿIlm al-Hisāb* was evaluated by G. Sobirov. However, this source has not been accessed either. G. P. Matviyevskaya and H. Tllashev, *Matematicheskiye i astronomicheskiye rukopisi uchonykh Sredney Azii X-XVIII* (Tashkent: 1981), 42.

16 Alī Qūshjī, *Risâle der İlm-i Hisâb,* Süleymaniye Manuscript Library, Ayasofya 2733/3, f. 170b-222a.

17 Muhammed Süveysî, "Hesap," in *TDV İslâm Ansiklopedisi*, accessed December 31, 2023, https://islamansiklopedisi.org.tr/hesap--matematik#1.

The same classification applies to the science of algebra. When *al-jabr* and *al-muqābala* were introduced to the world through al-Khwarizmi's canonical book *Kitāb al-Jabr wa'l-Muqābala*,[18] these new methods were initially regarded as calculation techniques. Although algebra later became established as an independent discipline, it is not uncommon to find sections on *al-jabr* and *al-muqābala* within arithmetic books.

Therefore, it would not be an exaggeration to state that arithmetic works constitute most of the Islamic mathematical literature. These works often combine Indian arithmetic, sexagesimal arithmetic, algebra, and mensuration. In addition to their theoretical significance, these texts also served practical purposes. Since al-Khwarizmī's time, algebra and *muqābala* were known to facilitate daily calculations related to commerce, *zakat* (almsgiving), inheritance, and land measurement. Likewise, mensuration played a crucial role in solving everyday measurement problems.

The most significant science employing arithmetic, algebra, and mensuration collectively is astronomy, which was the primary focus of the Samarkand school of astronomy and mathematics to which Alī Qūshjī belonged. Classical astronomy serves both as a theoretical approach to determining the positions, distances, and sizes of celestial spheres and bodies and as a practical discipline through *ilm al-mīqāt* (timekeeping science). For example, Qadi-zāde al-Rūmī's mathematical works—*Tuhfat al-Ra'is fī Sharh Ashkāl al-Ta'sīs*, *Risāla fī Istikhrāj Jayb Daraja Wāhida*, and *Hāshiya ʿala Tahrīr Usūl al-Handasa*—provide essential tools and methods for astronomical instruments and calculations. Similarly, Jamshīd al-Kāshī's *Miftāh al-Hisāb*, with its sections on sexagesimal and decimal arithmetic, is indispensable for astronomy, thus positioning it as a supplementary work to the field. However, these works also showcase the versatile nature of mathematics beyond its astronomical applications.

Alī Qūshjī's mathematical works, *Risāla dar ʿIlm-i Hisāb* and *al-Risāla al-Muhammadiyya*, should be viewed as both practical aids for daily calculations—ranging from commerce, inheritance, and *zakat* to land division—and as foundational texts providing the mathematical background necessary for astronomy. His approach of composing paired works in Persian and Arabic, first in mathematics and then in astronomy, reinforces this interpretation. This method aligns with the astronomy-focused nature of his scholarly tradition.

18 For a critical edition of *Kitāb al-Jabr wa'l-Muqābala*, see Roshdi Rashed, *Al-Khwārizmi: The Beginnings of Algebra* (London: SAQI, 2009).

After the dissolution of the vibrant mathematical environment in Samarkand following the consecutive deaths of Qadi-zāde al-Rūmī, al-Kāshī, and Ulugh Beg, Alī Qūshjī's journey from Tabriz to Istanbul played a pivotal role in transferring the mathematical and astronomical legacy of Samarkand to the emerging intellectual center of the Ottoman Empire. In Anatolia, the influence of the Maragha school was already widespread, with works like Nizām al-Dīn al-Nisābūrī's *al-Shamsiyya fī al-Hisāb*[19] and Ibn al-Hawwam's *al-Fawā'id al-Bahā'iyya fi'l-Qawā'id al-Hisābiyya*[20] (and its commentaries) familiarizing scholars with Central Asian mathematical knowledge. However, with Alī Qūshjī's arrival, both of his arithmetic works, alongside al-Kāshī's *Miftāh al-Hisāb*, became integral to the Ottoman *madrasa* curriculum, starting with the Aya Sofya and Fatih Madrasas.[21]

According to Cevat İzgi, before Bahā al-Dīn al-Āmilī's (d. 1031/1622) renowned *Khulāsā al-Hisāb* became widely circulated, *al-Risāla al-Muhammadiyya* and *Miftāh al-Hisāb* were the primary arithmetic textbooks in Ottoman *madrasa*s.[22] Although İzgi's claim has been widely accepted as a basic reference in modern scholarship, his conclusion regarding *al-Risāla al-Muhammadiyya* lacks clear evidence. The arguments based on manuscript numbers and geographical distribution are insufficient. The assumption that *Risāla dar 'Ilm-i Hisāb*, being in Persian, was not popular in the Ottoman realm, and that the Arabic *al-Risāla al-Muhammadiyya* was more widespread,[23] contradicts the manuscript evidence. In fact, the number of extant manuscripts of *Risāla dar 'Ilm-i Hisāb* found in libraries across Istanbul and Anatolia alone exceeds that of *al-Risāla al-Muhammadiyya* in the same region.

It is known that Hajjī Khalīfa began writing a commentary on *al-Risāla al-Muhammadiyya* titled *Ahsan al-Hadiyya bi-Sharh al-Risāla al-Muhammadiyya*, although this work remained incomplete, limited to an introduction. According to Ihsan Fazlıoğlu's article on Kâtip Çelebi's commentary, Hajjī Khalīfa stated that *al-Risāla*

19 *al-Shamsiyya fī al-Hisāb* was studied by Elif Baga as a master's thesis and later published by the Turkish Manuscripts Institution. Nizām al-Dīn al-Nisābūrī, *eş-Şemsiyye fî'l-Hisâb: Hesap Bilimlerinde Kılavuz,* ed. Elif Baga (İstanbul: Türkiye Yazma Eserler Kurumu Başkanlığı, 2020).

20 *el-Fevâ'idu'l-bahâ'iyye fî'l-kavâ'idu'l-hisâbiyye* was presented as a master's thesis by İhsan Fazlıoğlu at Istanbul University in 1993. İhsan Fazlıoğlu, "İbn el-Havvam ve Eseri el-Fevâid el-Bahâiyye fi el-Kavâidi el-Hisâbiyye: Tenkitli Metin ve Tarihi Değerlendirme" (Master's thesis, Istanbul University, 1993).

21 İhsan Fazlıoğlu, "Hesap," in *TDV İslâm Ansiklopedisi*, accessed December 29, 2023, https://islamansiklopedisi.org.tr/hesap--matematik#2-osmanlilarda-hesap.

22 İzgi, *Osmanlı Medreselerinde İlim,* 193.

23 İzgi, *Osmanlı Medreselerinde İlim,* 207.

al-Muhammadiyya was an abridgment of Ibn al-Hawwam's *al-Fawā'id al-Bahā'iyya* and al-Kāshī's *Miftāh al-Hisāb*.[24]

Considering the enduring fame of *Miftāh al-Hisāb* and the numerous accolades it has received, as well as its profound influence on Alī Qūshjī, it is not surprising to find potential similarities between the two works. However, definitive conclusions cannot be drawn without a detailed comparative analysis of their content, methodology, and style. Although existing sources have not conducted such a comparison for *Risāla dar ʿIlm-i Hisāb*, even a preliminary comparison of chapter titles suggests a comparable pattern.[25] Nonetheless, a thorough examination of both works is necessary to fully understand their relationship and the extent of their influence on each other.

2. *Risāla dar Ilm-i Hisāb*

The work titled "Treatise on the Science of Arithmetic" was authored by Alī Qūshjī during his time in Samarkand, sometime before the year 861 AH / 1457 CE.[26] The *Fankha Catalogue of Manuscripts* housed in Iranian libraries records 328 manuscript copies of this work.[27] In contrast, the *History of Ottoman Mathematical Literature* lists 66 copies, 28 of which are outside Iranian libraries.[28] Similarly, the same source records 16 entries for *al-Risāla al-Muhammadiyya*.[29] These figures provide insight into the widespread dissemination of *Risāla dar ʿIlm-i Hisāb* across Iran and Anatolia.

In this study, 26 manuscript copies have been examined, primarily from libraries in Türkiye, along with others from Tehran, Paris, Oxford, and Leiden. The catalog entries and, where available, the colophon dates of these manuscripts are presented in Table 1.

24 İhsan Fazlıoğlu, "Ali Kuşçu'nun el-Risâlet el Muhammediyye fi el-hisâb adlı eserine Kâtip Çelebî'nin yazdığı şerh: Ahsen el-hediyye bi-şerh el-Muhammediyye," *Türk Dilleri Araştırmaları* 17 (2007): 113–25.

25 For a critical edition of *Miftāh al-Hisāb*, see Jamshīd Giyās al-dīn al-Kāshī, *Miftāh al-Hisāb*, ed. Ahmad Said Demirdānis and Muhammad Hamdī Hanafī (Cairo: Dār al-Kitāb al-Arabī li al-tıbāʿa wa al-nashr, n.d.).

26 This information about the authorship is based on the earliest dated manuscript we have, which is registered under number 2640 in the Ayasofya collection at the Süleymaniye Library. This manuscript's transcription date is 861 AH / 1457 CE. Alī Qūshjī, *Risâle der İlm-i Hisâb,* Süleymaniye Manuscript Library, Ayasofya 2640/2, f. 170b-222a.

27 Derayāti, *Fankha*, 12:962–80.

28 İhsanoğlu et al., *Osmanlı Matematik Literatürü Tarihi*, 21–4.

29 İhsanoğlu et al., *Osmanlı Matematik Literatürü Tarihi*, 25.

2.1. Author's Manuscript

The author's manuscript of the work is located at the Süleymaniye Library, within the Ayasofya Collection, cataloged under number 2733, spanning folios 170b-222a of the codex.[30] The same codex also contains Alī Qūshjī's *al-Risāla al-Fathiyya fī al-Hay'a* on folios 1b-70a and *al-Risāla al-Muhammadiyya fī al-Hisāb* on folios 73b-168b.

It is recorded in various sources that Alī Qūshjī presented these works or the codex itself to Sultan Mehmed II. The title and colophon of *al-Risāla al-Fathiyya* explicitly indicates that the work was dedicated to the Sultan in commemoration of his victory at Otlukbeli. In *Kashf al-Zunūn*, Hajjī Khalīfa states that *al-Risāla al-Muhammadiyya* was written for Sultan Mehmed II.[31] Similarly, İhsan Fazlıoğlu notes that Alī Qūshjī, in the introduction of *al-Risāla al-Muhammadiyya*, explicitly mentions that he composed the work for presentation to the Sultan.[32]

A significant question that remains to be explored is whether the strong likelihood that *al-Risāla al-Fathiyya* and *al-Risāla al-Muhammadiyya* were presented to the Sultan also applies to the third work in the codex, *Risāla dar 'Ilm-i Hisāb*. To address this question, it is first necessary to determine whether the three works in the codex were brought together in the same period.

The pages of the first work, *al-Risāla al-Fathiyya,* feature occasional numbering in Arabic numerals. Some numbers appear incomplete, indicating that the codex has undergone restoration. These numbers do not appear in the other works. It is possible that the pages were trimmed during restoration, leading to the loss of numbering, or that only the first work was numbered initially. The numbering may have been added by the author himself. The handwriting in all three works is similar, the number of lines per page and the margin dimensions are consistent, and each work contains the author's colophon. Additionally, the catchwords on the verso sides of the folios have been consistently recorded in the same area of the page, suggesting that the author may have written the catchwords of all three works in succession, in a single sitting.

30 Alī Qūshjī, *Risâle der İlm-i Hisâb*, Ayasofya 2733/3, f. 170b-222a.

31 Kâtip Çelebi, *Keşfü'z-Zünûn*, Volume One, ed. Şerefettin Yaltkaya and Rıfat Bilge (İstanbul: Milli Eğitim Bakanlığı Yayınları, 1971), 889.

32 Fazlıoğlu, "er-Risâletü'l-Muhammediyye."

The colophon of *al-Risāla al-Fathiyya* records its completion date as Rabi'al-Awwal 878 AH (1473 CE). Alī Qūshjī dedicated this work to Sultan Mehmed II in celebration of his victory against Uzun Hasan at Otlukbeli, naming it *al-Risāla al-Fathiyya*. The second work in the codex, *al-Risāla al-Muhammadiyya*, was completed in Ramadan 877 AH (1473 CE). This work was named *al-Risāla al-Muhammadiyya* in honor of the Sultan. In the prefaces of both works, the Sultan's name is written in gold ink.

The completion date recorded in the colophon of *Risāla dar 'Ilm-i Hisāb* is Ramadan 877 AH (February 1473 CE), which coincides with the completion date of *al-Risāla al-Muhammadiyya*. However, unlike the other two works, this treatise lacks a preface. Moreover, while the titles of the first two works are listed on the endpaper of the codex, *Risāla dar 'Ilm-i Hisāb* is not mentioned. On folio 1a of the protective leaves, after the note "*al-Risāla al-Fathiyya qa gayruhu fî al-hisāb*," the titles *Risāla Fathiyya fî al-Hay'a* and *Risāla Muhammediyya fî al-Hisāb* appear, each followed by the name "Ali bin Muhammad al-Qūshī." The third entry, *Risāla al-Fārisī fî al-Hisāb*, lacks an author attribution. On folio 1a, the titles *Risāla Fathiyya fī İlm al-Hay'a* and *al-Risāla al-Muhammadiyya fī 'Ilm al-Hisāb* are found, but *Risāla dar 'Ilm-i Hisāb* is absent. These indications suggest that the third work may have been added to the codex at a later date.

Additionally, folio 1a bears the seal of Sultan Mahmud I, a foundation record, and the seal of Sheikhzādah Ahmed Efendi, Inspector of the Evkâf-ı Haramayn, along with the seal of Sultan Bayezid II. The third seal also appears at the end of the manuscript. Even if *Risāla dar 'Ilm-i Hisāb* was added later, the presence of Sultan Bayezid II's seal suggests that the codex existed in its present form during his reign.

The colophon of *Risāla dar 'Ilm-i Hisāb* is as follows (see Image 1):

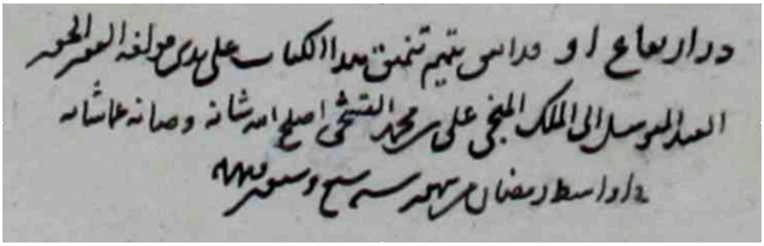


Image 1. The colophon of *Risāla dar ʿIlm-i Hisāb,* Ayasofya 2733/3.

قد اتفق تتميم و تنميق هذا الكتاب على يدي مؤلفه الفقير الحقير العبد المتوسل الى
الملك المنجي علي بن محمد القشجي أصلح الله شانه وصانه مما شأنه في أواسط
رمضان من شهور سنة سبع وسبعون وثمانمائة.[33]

"The completion and embellishment of this book were carried out by its author, the humble and destitute servant seeking refuge in the Savior, Ali bin Muhammad al-Qūshī—may God correct his affairs and protect him from that which may harm him—in the middle of the month of Ramadan in the year 877 AH."

Naturally, the existence of copies transcribed before this late authograph suggests that earlier authograph versions of the work must have existed. This suggestion raises the question of whether the work underwent modifications during its redaction—in other words, whether different versions of the text exist.

2.2. Versions of *Risāla dar ʿIlm-i Hisāb*

A comparative analysis of the manuscript copies used in this study reveals a noteworthy aspect: the authorial manuscript, cataloged as Süleymaniye Library Ayasofya 2733/3, exhibits syntactic differences from all other copies in certain instances. Specifically, the placement of the predicate varies between the two groups of manuscripts. In the authorial manuscript, the predicate appears at the end of the sentence, whereas in the other group, it is positioned at the beginning.

For example, on folio 179b of the Ayasofya 2733/3 manuscript (see Image 2):

33 Kuşçu, *Risâle der ʿİlm-i Hisâb,* 222a.

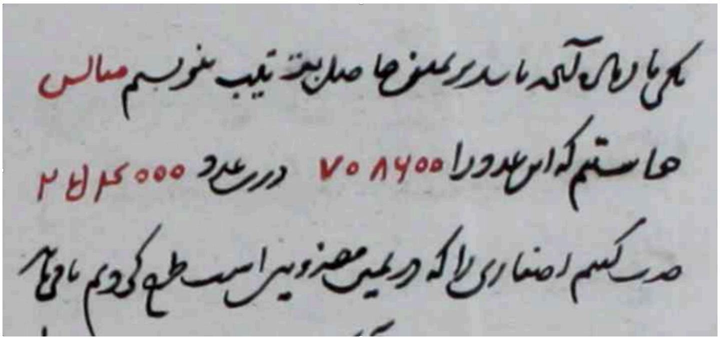


Image 2. Ayasofya 2733 – 179b

مثالش خواستیم که این عدد را ٧٠٨٦٠٠ درین عدد ٢٥٤٠٠٠ ضرب کنیم، اصفاري
را که بر یمین مضروبین است طرح کردیم.

Meanwhile, in folio 33b of the Süleymaniye Library Ayasofya 2640/2 manuscript, the same sentence with the same meaning is structured as follows (see Image 3):

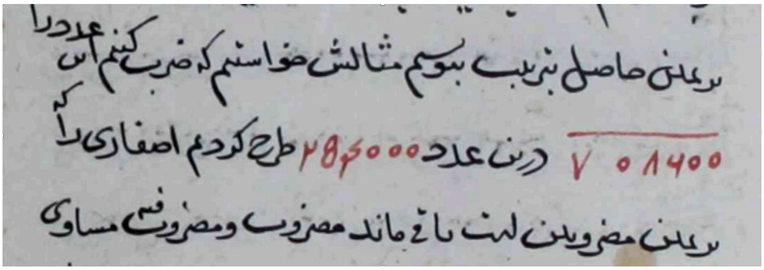


Image 3. Ayasofya 2640 – 33b

مثالش خواستیم که ضرب کنیم این عدد را ٧٠٨٦٠٠ درین عدد ٢٥٤٠٠٠، طرح
کردیم اصفاري را که بر یمین مضروبین است.

This variation in word order highlights the syntactic differences between the two manuscript groups.

Interestingly, this phenomenon is not observed in every sentence but only in certain instances. It is evident that when Alī Qūshjī revised his work in Istanbul, he made occasional syntactic modifications. Furthermore, in some folios of the manuscript found in the Ayasofya 2640 codex, the syntax was later corrected to align with that of the authorial manuscript (see Image 4).

An individual appears to have begun revising this earlier manuscript in accordance with the authorial version. However, the corrections were only made on two pages before the effort was abandoned.

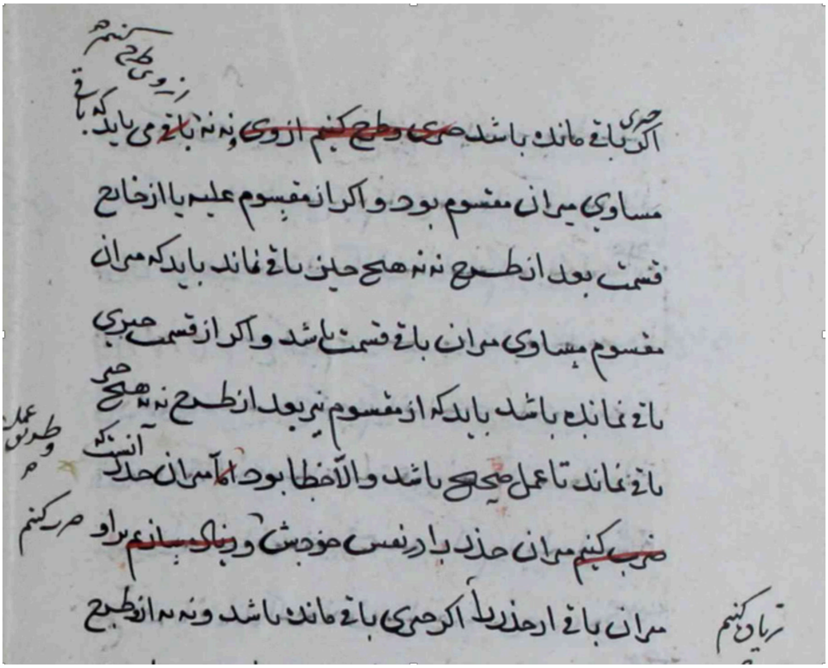


Image 4. Ayasofya 2640 – 40b

Since this difference is purely linguistic, it does not introduce any changes in meaning or content; therefore, it can be disregarded when considering the substance of the work. Different linguistic preferences may be observed across various geographical regions. The presence of inverted sentence structures in manuscripts other than the authorial copy, resembling spoken language, suggests the possibility that the work was compiled from a student's notes taken while listening to the author. It is likely that the initial version of the work originated from a student's notes after attending Alī Qūshjī's lectures. Regardless, this observation leads us to conclude that the first version of the text probably contained the syntactic structures found in manuscripts other than the authorial copy.

In addition to minor lexical variations frequently encountered in manuscript comparisons that do not alter the meaning, a significant content difference has also emerged. The second section of the first chapter, which deals with fractional arithmetic, discusses the concepts of inclusion, intersection, and co-primality among numbers and their common denominators. Based on the content of this section, the existing manuscripts can be classified into two groups. When considering the syntactic differences of the first group of manuscripts compared to the authorial copy, this content variation suggests that Alī Qūshjī updated his text even before the version he wrote upon arriving in Istanbul.

The translation of the relevant section from the authorial copy is as follows:

> The method is as follows: Take the denominators of the given fractions and simplify the larger of the denominators that have inclusion while retaining the smaller one. Record and set aside the remaining denominators sequentially. For example, take the largest denominator and examine the remaining ones; preserve those that are co-prime. For those that are compatible, simplify them by their common factor so that the remaining denominators settle at a fixed point.
>
> Then, among the remaining denominators, take the smaller of the ones that have inclusion and examine what remains. Retain the largest among them and preserve the co-prime denominators. For the compatible ones, take the common denominator. In this way, all denominators are reviewed.
>
> After the review, multiply one of the remaining denominators by the next, then multiply the result by the third, and repeat this process with the fourth. Apply this method to all denominators without exception. The final product will be the desired denominator.
>
> **Example**: If we seek the smallest number for nine fractions, we first take the denominators of these fractions. Since eight includes four and two, we eliminate them and record eight. Since three and six, as well as five and ten, have inclusion, we retain three and five, while recording six and ten. After examining the inclusion relationships, the remaining denominators are ten, nine, eight, seven, and six. Selecting one, say ten, and comparing it with the remaining denominators, we find that it is co-prime with nine and seven, so we record those numbers. Since eight and six are compatible by half, we take half of these values, i.e., four and three.
>
> Now, recording ten, we apply the same process to the remaining values—nine, seven, four, and three. Selecting nine and comparing it with the others, we see that it is co-prime with seven and four, so we record them. The included three is eliminated. Subsequently, since nine is co-prime with seven and four, we record both. Reviewing our recorded values, we find ten, nine, seven, and four. We multiply ten by nine, yielding ninety. Then, we multiply this result by seven, obtaining six hundred thirty. Finally, multiplying by four gives two thousand five hundred twenty. This is the desired result.[34]

In the second group of manuscripts, however, the passage is as follows:

34 Alī Qūshjī, *Risâle der İlm-i Hisâb*, Ayasofya 2733/3, f. 189a-190a.

> The method is as follows: Take the denominators of the given fractions and determine their inclusion, intersection, and co-primality. Now, preserve the co-prime denominators. For the denominators that have inclusion, retain the larger one and eliminate the smaller. Among the compatible denominators, retain one while preserving the proportional part of the other. Then, multiply the preserved values sequentially: First by the second, then by the third, and so on until all values have been considered. The final product will be the desired denominator.
>
> **Example**: Suppose we want to find the smallest common denominator of one-half, one-third, one-fourth, one-fifth, one-sixth, and one-eighth. The common denominator is sixty. That is, we take the denominators two, three, four, five, six, and eight. Since five is co-prime with each, we preserve it. As two and four are included in eight, we eliminate two and four while retaining eight. Between three and six, we keep six. Since six and eight are compatible, we preserve three as the proportional part of six and eliminate six.
>
> Now, reviewing the preserved numbers, we find three, five, and eight. Multiplying three by five gives fifteen. Multiplying this result by eight yields one hundred twenty, which is the desired denominator.[35]

As seen, in the first group, which includes the authorial copy, the method for finding the common denominator of fractions is explained in more detail and with a different approach than in the second group. Alī Qūshjī appears to have rewritten this section to be more comprehensive and explicit.

The authorial copy differs from the other manuscripts not only in syntax but also in its treatment of geometrical definitions. In the section on the elements of a circle, Alī Qūshjī made an additional revision, indicating a third update to his work. The relevant passage in the authorial copy is as follows:

> Each straight line is called a radius. Any line that divides the circle into two parts is called a chord, and each of these two parts is termed an arc. The plane enclosed by an arc and its corresponding chord is called a circular segment. The chord is referred to as the base of the segment. If the chord passes through the center, it is called a diameter.[36]

In manuscripts other than the authorial copy, this passage is given as:

> Each straight line is called a radius. Any line that divides the circle into two parts is called a chord, and each part is called an arc and a base. Each of these is termed a segment of the circle. Each of the two parts of the circumference is called an arc. If this line passes through the center, it is called a diameter.[37]

35 Alī Qūshjī, *Risâle der İlm-i Hisâb*, Ayasofya 2640/2, f. 42a-43a

36 Alī Qūshjī, *Risâle der İlm-i Hisâb*, Ayasofya 2733/3, f. 216a.

37 Alī Qūshjī, *Risâle der İlm-i Hisâb*, Ayasofya 2640/2, f. 67a.

The fundamental difference here lies in the definition of a circular segment, which is clearer and more precise in the authorial copy.

Ultimately, it can be concluded that the work exists in at least three versions. The first version, represented by the oldest known manuscript registered as Ayasofya 2640/2, contains both the original syntax and content. The second version, written in the older syntax but containing content updates, represents an intermediate revision. The final version, registered as Ayasofya 2733/3, includes both the updated syntax and the second round of content modifications. Manuscripts classified based on these distinctions are presented in Table 1.

	Library	Collection	Number	Copying dates (AH)
Version 1	Süleymaniye	Ayasofya	2640/2	861
	Süleymaniye	Ayasofya	2754/2	911
	Fransa Milli (BnF)	Supplement Persan	1757	983
	Süleymaniye	Hamidiye	875/1	1032
	Süleymaniye	Esmahan Sultan	295/2	1055
	Çorum İl Halk		3060/2	1067
	Süleymaniye	Laleli	56/7	1100
	Tahran Meclis-i Şûrâ-yı Millî		597/1	1158
	Süleymaniye	Kılıç Ali	683/4	11th Century
	Süleymaniye	Nuruosmaniye	2904/1	12th Century
	Millet	Ali Emirî	289	12th Century
	Arkeoloji Müzesi	Arkeoloji Müzesi	896/9	12th Century
	Beyazıt Devlet	Veliyüddin Efendi	2316/2	
	Raşid Efendi	Raşid Efendi	1119/5	
	Milli	Bes. A	4851/3	
	Tahran Meclis-i Şûrâ-yı Millî		206/2	
	Tahran Meclis-i Şûrâ-yı Millî		3117/4	
	Oxford University	Bodleian Greaves	21	
	Oxford University	Bodleian MS Laud Or.	313	
Version 2	Süleymaniye	Ayasofya	3336/3	9th Century
	Süleymaniye	Nuruosmaniye	4913/9	9th Century
	İstanbul Üniversitesi	Nadir Eserler	1082	9th Century
	Millet	Feyzullah Efendi	1337/3	10th Century
	Süleymaniye	Laleli	2766/3	10th Century
Version 3	Süleymaniye	Ayasofya	2733/3	878

Table 1. Manuscript Groups Based on the Examined Copies and Versions in the Study. For manuscripts with unknown copying dates, the estimated century information has been determined based on the relevant entry in the *Osmanlı Matematik Literatürü Tarihi*.[38] In Version 1, the syntax differs from that of the author's manuscript. Additionally, there are content differences in the sections on the inclusion (*tadāhul*), intersection (*tashāruk*), and co-primality (*tabāyun*) of numbers, as well as in the section on the elements of the circle. In Version 2, the syntax remains the same as in Version 1; however, the section on inclusion, intersection, and co-primality has been updated. In Version 3, both the syntax and the content of these sections have been revised and updated.

2.3. Analysis of Manuscript Copies and Versions

The differences we have identified indicate that none of the available copies were directly reproduced from the author's manuscript, which is recorded in Ayasofya 2733/3. It is evident that this manuscript, presented to the Sultan, remained in the palace library and did not circulate. Interestingly, except for the copy registered in Ayasofya 2640/2, all other copies were produced after the author's manuscript, yet they are not directly related to it. This lack of readership may also apply to other works in the collection, namely *al-Risāla al-Fathiyya* and *al-Risāla al-Muhammadiyya*.

The earliest dated copy, corresponding to Version 1, is found in a manuscript registered in the Süleymaniye Library's Ayasofya Collection under the number 2640. The colophon of *Risāla dar Ilm-i Hisāb* states that the manuscript was completed in the month of Ṣafar, 861 AH, by the scribe ʿAṭā'ullāh. Frequent corrections within the text and marginal annotations, including ellipsis marks, suggest that this copy was cross-checked against another source. However, no explicit comparison record is present.

None of the copies corresponding to Version 2 contain a recorded copying date. In the *Osmanlı Matematik Literatürü Tarihi*, approximate dates have been inferred based on the copying dates of other works within the same codices.

Among the examined codices, most contain treatises on astronomy, except for a few devoted exclusively to arithmetic and one covering various topics. In most of these manuscripts, works on astronomy appear first, followed by *Risāla dar Ilm-i Hisāb*. This suggests that ʿAlī Qūshjī's treatise on arithmetic was regarded as a preliminary study accompanying the science of astronomy.

38 İhsanoğlu et al., *Osmanlı Matematik Literatürü Tarihi*, 21–4.

2.4. Content of *Risāla dar ʿIlm-i Hisāb*

Risāla dar ʿIlm-i Hisāb consists of three articles. It does not contain a preface (*dibāja*) or an introduction (*muqaddima*). Instead, after a brief expression of praise (*ḥamdala* and *ṣalwala*), the text directly states, "This book consists of three articles," and then proceeds to the first section.

The first article concerns *Hindī Calculation* (Indian arithmetic), the second addresses *Tanjīm Calculation* (sexagesimal arithmetic), and the third and final article covers *Masāḥa* (mensuration). "*Hindī* Calculation" refers to computational methods based on the positional decimal numeral system. *Tanjīm* Calculation follows the sexagesimal numeral system, which was primarily used by astronomers. The study of mensuration (*masāḥa*), which involves the calculation of length, area, and volume of geometric objects, is commonly found in classical arithmetic texts.

The first article consists of an introduction and two chapters. In the introduction, the nine numerals used for representing numbers (*Hindī*-Arabic numerals) are introduced and how numbers are represented in the decimal positional system is explained. Chapter one discusses arithmetic operations involving whole numbers (only positive integers are considered in the text). It examines operations such as doubling, halving, addition, subtraction, multiplication, division, and root extraction. The treatise also describes methods for verifying these calculations.

Chapter two focuses on fractional arithmetic. It first explains how fractions are represented. Since addition and subtraction of fractions require a common denominator, the text first discusses methods for determining the greatest common divisor of whole numbers, followed by finding a common denominator. It also demonstrates the conversion between mixed fractions and improper fractions. Arithmetic operations on fractions, including doubling, halving, addition, and subtraction, are then covered. The text also explains unit conversions, exemplified through monetary units. The chapter concludes with multiplication, division, and root extraction of fractions.

The second article comprises an introduction and six chapters. In the introduction, firstly *Abjad* numerals and their use in numerical representation are introduced. Then the components of the sexagesimal (base-60) system, beginning with degrees and their subdivisions (minutes, seconds, thirds, etc.), as well as higher-order units (*marfūʿ*, *mathānī*, *mathālis*, etc.) are explained. Like the decimal system, a placeholder symbol is used in the sexagesimal system when a positional value is absent. The article frequently refers to "*Hindī* Calculation," highlighting key differences. The most evident difference is that in the sexagesimal system, numerical digits are written from right to left, rather than left to right as in *Hindī* Calculation.

In the following chapters, multiplication is introduced first, followed by division and root extraction. Additional sections are dedicated to determining the type of unit in computed results and verifying calculations. Other arithmetic operations are not detailed separately, as once the writing system of the sexagesimal system is understood, addition and subtraction can be performed analogously to *Hindī* Calculation.

The final chapter is on zodiacal calculation. In astronomical calculations, an additional unit called a *burj* (zodiac sign), equivalent to 30 degrees, is used. This section explains how arithmetic operations are performed when values include *burj* units.

The final article, devoted to *masāḥa*, consists of an introduction and three chapters. In the introduction the geometric entities that will be used, including points, lines, angles, surfaces, solids, two-dimensional and three-dimensional shapes, and their respective properties are defined. Chapter one covers length and area measurement for one-dimensional and two-dimensional geometric figures. Chapter two discusses the lateral and total surface areas of three-dimensional objects. Chapter three explains methods for calculating the volumes of solids.

It is worth noting that the *masāḥa* article is particularly concise. The authograph manuscript includes diagrams illustrating the geometric objects defined in the introduction, but no illustrations are provided in later sections. Some subsequently copied manuscripts, however, contain additional diagrams.

3. Comparison between *Risāla dar ʿIlm-i Hisāb and al-Risāla al-Muhammadiyya*

The comparative analysis of *Risāla dar ʿIlm-i Hisāb* and *al-Risāla al-Muhammadiyya* aims to shed light on the relationship between these two works, which constitutes one of this article's key research questions. To this end, Tables 2, 3, and 4 present a comparison of the relevant articles and chapter titles from both texts.

<table>
<tr><th colspan="3">Risāla dar ʿIlm-i Hisāb</th><th colspan="4">al-Risāla al-Muhammadiyya</th></tr>
<tr><td colspan="3"></td><td colspan="4">Introduction: The essence of number</td></tr>
<tr><td rowspan="22">Article 1: Hindī Calculation</td><td colspan="2">Introduction: Notations for numbers</td><td>Fann 1</td><td rowspan="20">Article 1: Hindī Calculation</td><td colspan="2">Chapter 1: Notations for numbers</td></tr>
<tr><td rowspan="8">Chapter 1: Arithmetic of whole numbers</td><td>1. Doubling</td><td rowspan="21"></td><td rowspan="8">Chapter 2: Arithmetic of whole numbers</td><td>1. Doubling</td></tr>
<tr><td>2. Halving</td><td>2. Halving</td></tr>
<tr><td>3. Summation</td><td>3. Summation</td></tr>
<tr><td>4. Subtraction</td><td>4. Subtraction</td></tr>
<tr><td>5. Multiplication</td><td>5. Multiplication</td></tr>
<tr><td>6. Division</td><td>6. Division</td></tr>
<tr><td>7. Root extraction</td><td>7. Root extraction</td></tr>
<tr><td>8. Checking</td><td>8. Checking</td></tr>
<tr><td rowspan="13">Chapter 2: Arithmetic of Fractional Numbers</td><td>Introduction</td><td rowspan="11">Chapter 2: Arithmetic of Fractional Numbers</td><td>Introduction</td></tr>
<tr><td>1. Finding common divisors</td><td>1. Notations for fractions</td></tr>
<tr><td>2. Finding common denominator</td><td>2. Finding common divisors</td></tr>
<tr><td>3. Transforming mixed fractions into improper fractions</td><td>3. Transforming mixed fractions and improper fractions one into another</td></tr>
<tr><td>4. Transforming improper fractions into mixed fractions</td><td>4. Finding common denominator</td></tr>
<tr><td>5. Doubling</td><td>5. Doubling, halving, summation, subtraction</td></tr>
<tr><td>6. Halving</td><td>6. Transforming compound fractions into proper or improper fractions</td></tr>
<tr><td>7. Summation</td><td>7. Multiplication</td></tr>
<tr><td>8. Subtraction</td><td>8. Division</td></tr>
<tr><td>9. Changing a unit to another unit</td><td>9. Root extraction</td></tr>
<tr><td>10. Multiplication</td><td>10. Transforming denominators one into another</td></tr>
<tr><td>11. Division</td><td></td><td></td><td></td></tr>
<tr><td>12. Root extraction</td><td></td><td></td><td></td></tr>
</table>

Table 2. Comparison of the first articles *Risāla dar ʿIlm-i Hisāb* and *al-Risāla al-Muhammadiyya*

al-Risāla al-Muhammadiyya		*al-Risāla al-Muhammadiyya*	
Article 2: *Tanjīm* Calculation	Introduction: Abjad notation and some definitions	Article 2: *Tanjīm* Calculation	Introduction: Abjad notation and some definitions
	1. Grid multiplication		1. Doubling, summation, subtraction
	2. Division		2.Multiplication
	3. Root extraction		3. Division
	4. Multiplication, division and knowing the type of the result		4. Root extraction
	5. Checking		5. Checking
	6. Zodiac signs calculation		

Table 3. Comparison of the second articles *Risāla dar ʿIlm-i Hisāb* and *al-Risāla al-Muhammadiyya*

Risāla dar ʿIlm-i Hisāb		*al-Risāla al-Muhammadiyya*	
Article 3: *Masāha*	Introduction: Definitions	*Fann 2: ʿIlm-i Masāha*	Introduction: Definitions
	1. Mensuration of lines and plane surfaces		Chapter 1: Mensuration of lines and plane surfaces
	2. Mensuration of surface of revolution		Chapter 2: Mensuration of surface of revolution
	3. Mensuration of volumes		Chapter 2: Mensuration of volumes

Table 4. Comparison of the third article *Risāla dar ʿIlm-i Hisāb* and the second *fann al-Risāla al-Muhammadiyya*

In *al-Risāla al-Muhammadiyya*, the subjects of algebra and the double false position method as well as some algebra problems, which are examined in the third, fourth, and fifth articles of the first section (*fann*), do not have corresponding counterparts in *Risāla dar ʿIlm-i Hisāb*. Therefore, these topics have been excluded from the comparison.

Analyzing the chapter titles of a work often falls short of providing definitive conclusions regarding its content. However, given that both texts were authored by the same scholar, a degree of overlap in content can be expected. In this specific case, the comparative tables above serve as an initial step in forming an understanding of the relationship between the two texts.

As observed, the chapter titles in the first article, which examines Indian arithmetic, remain nearly identical despite some variations in sequencing. Upon closer examination, the content of this section exhibits significant similarities. However, the situation differs in the second article, which focuses on *tanjīm* arithmetic. While *Risāla dar ʿIlm-i Hisāb* discusses only multiplication, division, and root extraction, *al-Risāla al-Muhammadiyya* expands upon these topics by including methods for doubling, halving, addition, and subtraction. Furthermore, *al-Risāla al-Muhammadiyya* integrates zodiacal calculations within the main discussion, rather than addressing them under a separate heading. A detailed textual comparison reveals that the second articles of each work do not maintain the level of similarity observed in the first articles, and this difference also applies to the third article. Although the chapter titles align, the textual content diverges considerably.

Additionally, *al-Risāla al-Muhammadiyya* contains three extra sections: one devoted to algebra (*ilm al-jabr*), which discusses equation-solving methods; one covering the double false position method, a distinct approach to solving equations; and another containing some algebraic problems. *Risāla dar ʿIlm-i Hisāb* lacks corresponding counterparts for these sections. These three additional articles, in fact, introduce more advanced computational techniques.

A crucial distinguishing feature of *al-Risāla al-Muhammadiyya* is its introduction, which includes a discussion of the nature of numbers. This discourse, commonly found in classical mathematical treatises, provides insight into the author's conceptualization of numbers. However, *Risāla dar İlm-i Hisāb* does not include such a discussion, raising questions as to why the author omitted this segment.

Another major difference between the two works lies in their technical level. Although *Risāla dar ʿIlm-i Hisāb* covers the three primary computational methods of its time—Indian arithmetic, sexagesimal arithmetic, and *masāḥa* (mensuration)—it presents these topics at an introductory level. In contrast, *al-Risāla al-Muhammadiyya* extends its scope by introducing algebra and the double false position method. While its sections on Indian and sexagesimal arithmetic are more concise than those in *Risāla dar ʿIlm-i Hisāb*, its broader treatment of *masāḥa* and inclusion of new mathematical discussions distinguish it as a more advanced work.

It is important to note that there is no historical evidence—either from the author's own statements or from later classical scholars—supporting the claim that *Risāla dar ʿIlm-i Hisāb* is a translation or an expanded version of *al-Risāla al-Muhammadiyya*. The argument favoring translation, found in modern literature, appears to stem from the significant textual similarities between the respective first articles on Indian arithmetic. However, this resemblance is not sustained throughout the rest of the text.

In light of the comparative findings summarized above, the assertion that *al-Risāla al-Muhammadiyya* is a translation of *Risāla dar ʿIlm-i Hisāb* loses its validity. The claim that the former represents an expanded version also remains debatable, as the similarities are largely confined to the first article. Furthermore, the section on sexagesimal arithmetic exhibits notable differences in exposition, and the additional three articles in *al-Risāla al-Muhammadiyya* represent original contributions rather than expansions of existing topics. Consequently, it is more appropriate to consider these works as independent compositions.

The more comprehensive content of *al-Risāla al-Muhammadiyya*, its discussion of the nature of numbers, and its additional sections suggest an evolution in the author's approach to mathematics and mathematics education over time. Moreover, the fact that *Risāla dar ʿIlm-i Hisāb* was included in the same manuscript codex as *al-Risāla al-Fathiyya*, a work presented to the Sultan following the Ottoman victory at Otlukbeli, indicates the author's intention for the former text to remain in circulation. It is likely that the author prepared these two works for different audiences, which may explain why he did not consider *Risāla dar ʿIlm-i Hisāb* obsolete after composing *al-Risāla al-Muhammadiyya*. The large number of surviving copies of *Risāla dar ʿIlm-i Hisāb*, copied and reproduced across different centuries, further supports this interpretation.

4. The Concept of Root Extraction in *Risāla dar ʿIlm-i Hisāb*

One of the frequently encountered topics in classical arithmetic treatises is the approximation of the square root of integers. Depending on the level of the text, different degrees of root extraction may be addressed. Given that *Risāla dar ʿIlm-i Hisāb* is a fundamental arithmetic work, Alī Qūshjī limited his discussion to the extraction of square roots in both the *Hindī* and *Tanjīm* arithmetic sections.

In this section, as an example of the work's content, the method for determining the square root of integers will be examined. In addition to presenting the traditional explanation found in the text, a mathematical proof will be provided to demonstrate why the method yields the correct result.

A number, when multiplied by itself, produces a product known as its *square*. Given a number, the value that, when squared, results in this number, is called its *square root*.

For $a \geq 0$;

$$a \times a = a^2$$

For a given number *a* and another number *b*, if:

$$b^2 = a$$

then *b* is defined as the square root of *a*. The square root is denoted by the symbol √.

4.1. The Method of Extracting Square Roots[39]

The process of root extraction is performed with the aid of a table. The number whose square root is to be found is written into the table, with each digit occupying a separate column. Starting from the rightmost digit, every other digit (first, third, fifth, etc.) is marked with dots.

A digit between 1 and 9 is identified such that, when squared, it yields the largest possible value that can be subtracted from the number formed by the marked digit and those to its left. This selected digit is then written above the corresponding mark and again below it, leaving appropriate spacing. The square of this digit is calculated and recorded beneath the original number, aligning its units digit with the corresponding digit in the original number. The square is then subtracted from the aligned digits of the original number, and a horizontal line is drawn to separate the remainder, which is written below.

The number written above is then doubled, and its sum is shifted one place to the right so that its units digit aligns with the digit immediately to the right of the last marked position. A horizontal line is drawn over the lower number to indicate completion of that step.

The process is repeated for the next marked position: a digit from 1 to 9 is chosen such that, when squared, it yields the largest possible value that can be subtracted from the number formed by the marked digit and those to its left. This digit is recorded above and below the corresponding mark, and the previously mentioned steps are repeated. Each time, the sum of the numbers written above is shifted one place to the right.

39 This method is explained in Alī Qūshjī, *Risâle der İlm-i Hisâb*, Ayasofya 2733/3, f. 183a-186a.

The steps continue in this manner until the leftmost marked position is reached. If at any stage no suitable digit can be found, zero is written above and below the mark, and the sum is shifted one place to the right.

Upon completion of the process, the number recorded at the top of the table represents the extracted square root. If no remainder remains, the original number is a perfect square, and the obtained value is its exact square root. If a remainder is present, the number is irrational, and the result represents an approximation. In such cases, the rightmost marked digit at the top is added to itself, and the sum of all lower numbers is considered the denominator. The ratio of the original number to this sum is computed, and the resulting fraction is appended to the extracted root for improved precision.

4.2. Algebraic Explanation of the Method

The rule applied in the author's classical method can be expressed algebraically as follows:

Let x be the natural number whose square root is to be found.

$$x=u^2+r$$

where u and r are numbers that satisfy this equation. Let u be chosen as the largest possible number. If x is a perfect square, then

$$x=u^2$$

Let u be written as follows, where $0 \leq a_1,\ldots,a_{n-1} \leq 9$ and $0<a_n \leq 9$:

$$u=10^{n-1} \times a_n + 10^{n-2} \times a_{n-1} + \ldots + a_1$$

In this case,

$$x=(10^{n-1} \times a_n + 10^{n-2} \times a_{n-1} + \ldots + a_1)^2 + r$$

$$= (10^{n-1})^2 a_n^2 + (10^{n-2})^2 a_{n-1}^2 + \ldots + a_1^2$$

$$+2(10^{n-1} \times a_n)(10^{n-2} \times a_{n-1} + \ldots + 2(10^{n-1} \times a_n) a_1$$

$$+2(10^{n-2} \times a_{n-1})(10^{n-3} \times a_{n-2} + \ldots + 2(10^{n-2} \times a_{n-1}) a_1$$

$$+ \ldots + 2 \times 10 \times a_2 a_1$$

That is, the number x is equal to the sum of the squares of each term in

$$u=10^{n-1}\times a_n+10^{n-2}\times a_{n-1}+\ldots+a_1$$

along with twice the product of each pair of terms.

The square root extraction method relies on estimating the values of a_i in this sum, subtracting their squares and the twice-their-product terms one by one from x. If a remainder remains at the end of these subtractions, this remainder is divided by $2u+1$, and the resulting fraction is added to u. Thereby, an approximate square root is obtained. In other words,

$$x=\sqrt{u^2+r}\approx u+\frac{r}{2u+1}$$

Now, we will apply this rule to the example in the work: finding the square root of 128172.

Since the number has six digits, its square root cannot have more than three digits.

$$128172=(10^2a_3+10a_2+a_1)^2+r$$

$$=10000a_3^2+100a_2^2+a_1^2+a_3\times a_2$$

$$+200\times a_3\times a_1$$

$$+20\times a_2\times a_1+r$$

To subtract the largest possible $10000a_3^2$ value from 128172, we choose $a_3=3$.

$$128172-90000=38172$$

From 38172, we subtract the largest possible $100a_2^2$ value. However, we must also keep in mind that the term

$$2000\times a_3\times a_2=6000\times a_2$$

will be substracted. Thus, we choose $a_2=5$.

$$38172-6000\times 5-100\times 25=38172-30000-2500=5672$$

Now, from 5672, we subtract the largest possible a_1^2 value. However, we must also consider that the terms $200\times a_3\times a_1=600\times a_1$ ve $20\times a_2\times a_1=100\times a_1$ will be substracted.

So, we choose $a_1=8$.

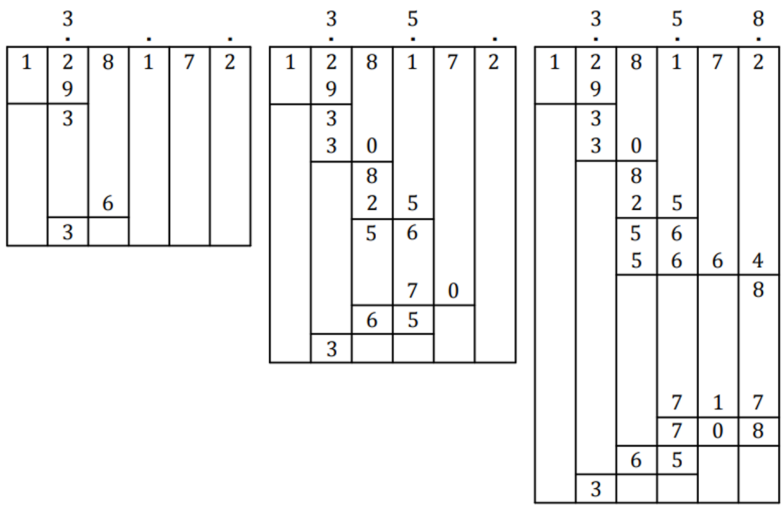
$$5672 - 600 \times 8 - 100 \times 8 - 64 = 5672 - 4800 - 800 - 64 = 8$$

Thus, we find u= 385 and r= 8 .

$$128172 = 358^2 + 8$$

$$\sqrt{128172} \approx 358 + \frac{8}{717}$$

The tables provided in the work for this example, using Western Arabic numerals, are as follows:



4.1. The Persian Text of the Square Root Extraction Method

In this section, the original Persian text of the square root extraction method, whose translation and algebraic explanation were given above, will be provided based on the author's manuscript registered as Ayasofya 2733.

[183a][40]

فصلِ هفتم: در استخراجِ جذر

هر عددی که او را در نفسِ خودش ضرب کنند، آن عدد را جذر خوانند و حاصلِ ضرب را مجذور و مربّع و مال.

طریقِ عملِ جذر آن است که عددی را که جذرِ او مطلوب باشد، بر جایی نویسیم. و بر بالائ او خطِّ عرضی کشیم، هم چنانکه در عملِ قسمت گفتیم. و به نقطها نشان کنیم بر خطِّ عرضی برابرِ مراتبِ فرد؛ مثل مرتبهٔ آحاد که اوّل است، و مآت که سیّم است، و عشرات الوف که پنجم است و علی هذا آنچه باشد.

و اکثر عددی طلب کنیم از آحاد که مضروبِ او را در نفسِ خودش از ما یحاذي علامتِ اخیره به صورتش و از یسارش اگر در یسارش چیزی باشد، نقصان توان کرد. هر چه گاه همچنین عدد یافت شود، او را بر بالائ علامتِ اخیره نویسیم و در تحتِ علامت نیز نویسیم همین عدد را [183b] به مسافتئ مناسب در محاذاتِ او. عددِ فوقانی را در عددِ تحتانی ضرب کنیم، یعني در نفسِ خودش. و حاصل را در تحتِ عددی که جذرِ او مطلوب است نویسیم، چنانچه آحادش محاذيء مضروب فیه واقع شود. و او را از ما یحاذي مضروب فیه و از یسارِ او نقصان کنیم. و باقي را در تحتِ خطِّ عرضی نویسیم. بعد از آن فوقانی را بر تحتانی افزاییم، و مجموع را به جانبِ یمین به یك مرتبه نقل کنیم، چنانکه آحادش محاذي یمینِ علامتِ اخیره شود.

بعد از آنکه خطِّ عرضی بر فوقِ رقمِ تحتانی به جهتِ محو کشیده باشیم. باز طلب کنیم اکثر عددی از آحاد که چون او را در نفسِ خودش ضرب کنیم، و در مجموعِ منقول نیز ضرب کنیم، ممکن باشد طرحِ او از صورتِ عددی که در محاذاتِ علامتِ مقدّم بر علامتِ اخیره است و از آنچه در یسارِ اوست. هر چه گاه این چنین عدد یافت شود، بر بالائ علامتِ مقدّم بر علامتِ اخیره [184a] نویسیم. و همچنین در تحتِ او نیز نویسیم. و عملِ مذکور به جاي آریم. بعد از آن عددِ فوقانی را بر تحتانی افزاییم، و این مجموع را با مجموعِ اوّل به جانبِ یمین نقل کنیم.

40 Metinde varak numaraları Ayasofya 2733'e kayıtlı müellif nüshasına göre verilmiştir.

باز طلب کنیم اکثر عددی از آحاد که چون در نفسِ خودش و در مجموعِ منقول ضرب کنیم، ممکن باشد طرحِ آن از صورتِ عددی که در محاذاتِ علامتِ مقدّم بر آن دو علامتِ مذکوره باشد و از آنچه در یسارِ او نیز باشد. هر چه گاه که این چنین عدد یافتیم، با او عملِ سابق به جايِ آریم. و اگر اینچنین عدد نیابیم، بر فوقِ علامت و تحتش صفری نویسیم. و مجموعاتِ مذکوره را به يك مرتبه به جانبِ یمین نقل کنیم. و همچنین عمل می کنیم تا منتهی شود به علامتِ اوّل. با او نیز همین عمل به جايِ آریم.

پس آنچه حاصل شود بر فوقِ جدول جذر باشد آن عددی را که مطلوب بود جذرِ او. و اگر چیزی باقي نماند در صفِّ عدد، این عدد [184b] منطق الجذر باشد، و ارقامی که بر بالائ خطِّ عرضی است جذرِ او باشد به تحقیق. و اگر چیزی ماند، معلوم شود که او اصمّ الجذر بوده است. پس یکی را با آنچه بر بالای علامتِ ایمن واقع است بر آنچه در تحتِ او واقع است افزاییم. و جمیع عددِ تحتانی را مخرج فرض کنیم. و باقي از عددی که جذرِ او مطلوب است به این مجموع نسبت کنیم. آنچه حاصل شود بر بالای علامات با این کسر جذرِ عددِ مذکور بود تقریب اصطلاحي.

مثالش خواستیم که این عدد را ۱۲۸۱۷۲ جذر استخراج کنیم، به همان طریق قسمتِ او را نوشتیم و خطِّ عرضی و دیگر خطوطِ طولی بر کشیدیم. و علامات چنانکه گفتیم تعیین کردیم.

بعد از آن اکثر عددی طلب کردیم به صفتِ مذکوره. عددِ سه را یافتیم. این را بر فوقِ علامتِ اخیره و در تحتش به مسافتئ مناسب نوشته، فوقانی را در تحتانی ضرب کرده، حاصل را که نه است در تحتِ دو که برابرِ سه است [185a]نوشته، از او و از یسارِ او نقصان کردیم. و باقي را که سه است بعد از خطِّ عرضی در برابرِ دو نوشتیم. پس فوقانی را بر تحتانی افزودیم، و مجموع را که شش است يك مرتبه به جانبِ یمین نقل کردیم. بعد از آنکه خطِّ عرضی بر فوقِ سهٔ تحتانی کشیدیم. بر این صورت:

	٣				
	.		.		.
١	٢	٨	١	٧	٢
	٩				
	٣				
		٦			
	٣				

باز اکثر عددئ دیگر به صفتِ مذکور طلب کردیم، پنج را یافتیم. او را بر بالای علامتی که مقدّم است، بر علامتِ اخیره و در تحتِ یمینِ علامت بر یمبنِ آحادِ منقول یعنی شش نوشتیم. و پنج را اوّلا در شش ضرب کردیم، سی حاصل شد. آن را در تحتِ عددِ مجذور نوشتیم چنانچه صفر در برابرِ شش افتاد. پس او را از ما یحاذئ او از عددِ مجذور نقصان کردیم، هشت باقي [185b]ماند. آن را در تحتِ صفر نوشتیم بعد از خطِّ عرضی. بعد از آن پنج را در پنجِ تحتانی ضرب کرده، حاصل او را که بیست و پنج است، به صفتِ مذکور نوشته، از ما یحاذئ او نقصان کردیم؛ پنجاه و شش باقي ماند. آن را بعد از خطِّ عرضی نوشتیم. پس پنجِ فوقانی را با پنجِ تحتانی جمع کردیم، ده شد. صفر به جای پنجِ تحتانی اعتبار کرده، یکی بر شش که بر یسارِ اوست افزودیم. و مجموع را یك مرتبۀ دیگر به جانبِ یمین نقل کردیم بعد از تخطیطِ آنچه بیشتر بود در سطرِ تحتانی. بر این صورت:

	٣		٥		
	.		.		.
١	٢	٨	١	٧	٢
	٩				
	٣				
	٣	٠			
		٨			
		٢	٥		
		٥	٦		
			٧	٠	
		٦	٥		
	٣				

باز اکثر عددی به صفتِ مذکور طلب کردیم، هشت را یافتیم. او را بر بالای علامتِ اولی و در تحتِ او، بر یمینِ صفرِ تحتانی نوشتیم. و این هشت را اوّلا در هفت ضرب کردیم. و حاصلِ ضرب را از ما یحاذئ او نقصان کردیم، هیچ نماند. و بعد از آن [186a] در هشت ضرب کردیم. و حاصل را از آنچه در محاذاةِ مضروب فیه و از یسارِ اوست نقصان کردیم. پس باقي ماند از عددِ مجذور هشت. بعد از آن هشتِ فوقانی را با هشتِ تحتانی جمع کرده یکی بر او افزودیم، عددِ تحتانی هفتصد و هفده شد، و عمل تمام شد. بر این صورت :

	٣		٥		٨
١	٢	٨	١	٧	٢
	٩				
	٣				
	٣	٠			
		٨			
		٢	٥		
		٥	٦		
		٥	٦	٦	٤
					٨
			٧	١	٧
			٧	٠	٨
		٦	٥		
	٣				

و این هفتصد و هفده مخرجي است که هشت باقي کسرِ اوست به تقریب. پس جذرِ حاصل از عمل باشد این:

٣٥٨
٨
٧١٧

5. Conclusion

In contemporary literature regarding *Risāla dar ʿIlm-i Hisāb*, a commonly circulated claim—propagated through the statements of historians of science such as Adnan Adıvar, Salih Zeki, and Cevat İzgi—asserts that *al-Risāla al-Muhammadiyya* is either a translation or an expanded version of the former work. This translation claim may have contributed to the assumption that *Risāla dar ʿIlm-i Hisāb* lost significance following the compilation of *al-Risāla al-Muhammadiyya.* The fact that the latter was written around the time Alī Qūshjī arrived in Istanbul may have led to the perception that it was the more widely circulated work in the Ottoman world. İzgi, who proposed this claim, supports his argument by comparing the number of existing manuscripts and their geographical distribution. However, although İzgi's manuscript counts are outdated, even according to his data, the copies of *Risāla dar ʿIlm-i Hisāb* found in Anatolia alone exceed the total copies of *al-Risāla al-Muhammadiyya*. Furthermore, more recent cataloging efforts reveal nearly four hundred manuscripts *Risāla dar ʿIlm-i Hisāb* scattered across Iran, Turkey, France, England, the Netherlands, and several other countries. Based on numerical comparison alone, *Risāla dar ʿIlm-i Hisāb* clearly surpasses the latter work, suggesting that the widespread acceptance of this translation claim requires further substantiation. Until more concrete evidence is presented, it is advisable to approach İzgi's assertion with caution.

Another claim that requires substantiation is the idea that Alī Qūshjī translated his Persian work into Arabic upon or shortly after his arrival in Istanbul, thereby producing *al-Risāla al-Muhammadiyya*. The scholars cited in this claim—Adıvar and Salih Zeki—likely reached this conclusion based on the thematic alignment of the works and the similarity of the first chapters of the two works, which deal with whole numbers and fractions. However, Alī Qūshjī himself does not explicitly mention translation or expansion as an objective. As demonstrated in this study, although the titles align, methodological and content differences emerge in sections such as *Tanjīm* calculation (base-60 arithmetic) and *masāḥa* (mensuration). Furthermore, Alī Qūshjī opted not to include algebra and the double false position method in *Risāla dar ʿIlm-i Hisāb*. The discussion of the nature of numbers found in the introduction of *al-Risāla al-Muhammadiyya* is likewise absent from the former work. These distinctions indicate that rather than expanding upon his initial work, the author intended to compose a second mathematical treatise at a different level. This conclusion not only clarifies the relationship between the two texts but also highlights the necessity of re-evaluating previous literature regarding historical texts. Particularly, claims of translation should not be made without direct examination of the content.

A comparative analysis of content and methodology suggests that Alī Qūshjī wrote his two treatises on arithmetic for different audiences. The inclusion of *Risāla dar ʿIlm-i Hisāb* in various manuscript codices after the composition of *al-Risāla al-Muhammadiyya* indicates that the author still regarded the former work as valuable. Another potential reason for its continued significance is its sufficiency for mathematical requirements in astronomy. Indeed, in most examined manuscript codices, *Risāla dar ʿIlm-i Hisāb* appears alongside various astronomical works, whereas this is not the case for *al-Risāla al-Muhammadiyya*, except for the author's compiled manuscript. This distinction supports the argument that *Risāla dar ʿIlm-i Hisāb* was traditionally viewed as an introduction to astronomy. A crucial step toward definitively proving this hypothesis would involve comparing the mathematical content of *Risāla dar ʿIlm-i Hisāb* with the mathematical applications in astronomical works such as Ulugh Beg's *Zīj* and *al-Risāla al-Fathiyya*.

While this study suggests that Alī Qūshjī primarily composed *Risāla dar ʿIlm-i Hisāb* as a guide to astronomy, it is evident that the treatise also serves as a comprehensive arithmetic manual. Sections covering Indian numerals, *tanjīm* calculation, and *masāḥa* contain all the necessary methods for daily calculations. The text's pedagogical approach is evident in both its structure and examples. The explanations are clear and direct, while the examples are practical, appropriately challenging, and sufficient in number. Consequently, it is apparent that the work functioned as a fundamental arithmetic book independent of astronomy. Its continued use in both Anatolia and Iran until the twentieth century underscores its resilience and enduring success.

One of the challenges encountered in this study is the sheer number of manuscript copies. Out of 356 known copies, predominantly housed in Iran, only 26 were accessible for this research. Notably, the inclusion of a manuscript authored by Alī Qūshjī in the final years of his life proved to be a fortunate discovery. Since the treatise was originally written in Samarkand and had already spread across a vast region before his arrival in Istanbul, obtaining a copy penned by the author himself provides the most complete and updated version of the text. A comparison of the authograph with other copies reveals differences in sentence structure, indicating that no existing copies were directly transcribed from this version. This result suggests that the codex prepared for the Sultan remained within the palace library and did not circulate widely. Further comparisons reveal textual variations between different copies. For instance, the section on finding common denominators of fractions differs in some versions from the autograph. Similarly, in the part explaining the elements of a circle, the autograph contains additional sentences not found in any other copy. These findings indicate that Alī Qūshjī revised his work three times. Initially, he authored the treatise in Samarkand with an older style and content. Later, between Samarkand and Istanbul, he updated the section on common denominators. Finally, upon reaching Istanbul, he modified the sentence structure and expanded the section on the elements of a circle. These modifications aimed to enhance clarity and completeness.

The fact that Alī Qūshjī revised his work multiple times serves as a critique of research methodologies that rely solely on a single manuscript copy. The version of *Risāla dar ʿIlm-i Hisāb* compiled in Istanbul in 877 AH (1473 CE) represents its most updated form. Having this latest version at hand is advantageous compared to relying on earlier versions. However, given that more than 300 copies are housed in Iranian libraries, further research may uncover additional authographs or evidence supporting earlier versions.

References

Adıvar, Adnan. *Osmanlı Türklerinde İlim*. Vol. 4. Baskı. İstanbul: Remzi Kitabevi, 1982.

Ali Kuşçu. *Astronomiya oid risola*. Translated by I. M. Muminov. Taşkent: Uzbekiston Ssr «Fan» Našriëti,1968.

———. *er-Risâletü'l-fethiyye fî'l-hey'e*. Süleymaniye Manuscript Library, Ayasofya 2733/3, v. 1b-70a;

———. *er-Risâletü'l-Muhammediyye fî'l-hisâb*. Süleymaniye Manuscript Library, Ayasofya 2733/3, v. 73b-168b

———. *Risâle der İlm-i Hisâb*.

Millet Kütüphanesi, Feyzullah Efendi 1337/3.

Süleymaniye Manuscript Library, Ayasofya 2640/2.

Süleymaniye Manuscript Library, Ayasofya 2733/3.

Baga, Elif. "Osmanlı Klasik Dönemde Cebir." PhD diss., Marmara University, 2012.

Bilgin, Zehra. "Hesab Bilimine Giriş: Ali Kuşçu'nun *Risâle der İlm-i Hisâb* Adlı Eseri- Tenkitli Metin, Çeviri, Değerlendirme-." PhD diss., İstanbul Medeniyet University, 2024.

Derayâti, Mustafa. *Fankha* (*Fihrist Nusakh Khatti Iran, Iranian Manuscripts Index*). Tehran: National Library of Iran, 1391.

Fazlıoğlu, İhsan. "Ali Kuşçu'nun bir hendese problemi ve Sinan Paşa'ya nisbet edilen cevabı." *Divan: Disiplinlerarası Çalışmalar Dergisi* 1 (December 1996): 85–106.

———. "Ali Kuşçu'nun el-Muhammediyye fî el-hisâb'ının 'Çift yanlış' ile 'Tahlîl' hesâbı bölümü." *Kutadgubilig* 4 (October 2003): 135–55.

———. "Hesap." In *TDV İslâm Ansiklopedisi*. accessed December 29, 2023. https://islamansiklopedisi.org.tr/hesap--matematik#2-osmanlilarda-hesap.

———. "İbn el-Havvam ve Eseri el-Fevâid el-Bahâiyye fi el-Kavâidi el-Hisâbiyye Tenkitli Metin ve Tarihi Değerlendirme." Master's thesis, İstanbul University, 1993.

———. "Osmanlı Felsefe-biliminin Arkaplanı: Semerkand Matematik Astronomi Okulu." *Dîvân: Disiplinlerarası Çalışmalar Dergisi* 14 (June 2005): 1–66.

———. "er-Risâletü'l-Muhammediyye." In *TDV İslam Ansiklopedisi*. accessed December 31, 2023. https://islamansiklopedisi.org.tr/er-risaletul-muhammediyye.

———. "The Samarqand Mathematical and Astronomical School A Basis for Ottoman Philosophy and Science." *Journal for the History of Arabic Science* 14 (2008): 3–68.

———. "Ali Kuşçu'nun el-Risâlet el Muhammediyye fi el-hisâb adlı eserine Kâtip Çelebî'nin yazdığı şerh: Ahsen el-hediyye bi-şerh el-Muhammediyye." *Türk Dilleri Araştırmaları* 17 (2007): 113–25.

İhsanoğlu, Ekmeleddin vd. *Osmanlı Matematik Literatürü Tarihi*. Vol. 1. İstanbul: IRCICA Yayınları, 1999.

İzgi, Cevat. *Osmanlı Medreselerinde İlim.* İstanbul: Küre Yayınları, 2019.

Jamshīd Giyās al-dīn al-Kāshī, *Miftāh al-Hisāb*. Edited by Ahmad Said Demirdānis and Muhammad Hamdī Hanafī. Cairo: Dār al-Kitāb al-Arabī li al-tıbā'a wa al-nashr, n.d.

Kâtip Çelebi. *Keşfü'z-Zünûn*, Volume One. Edited by Şerefettin Yaltkaya and Rıfat Bilge. İstanbul: Milli Eğitim Bakanlığı Yayınları, 1971.

Matviyevskaya, G.P. and H. Tllashev. *Matematicheskiye i astronomicheskiye rukopisi uchonykh Sredney Azii X-XVIII.* Taşkent: Izdatel'stovo «Fan» Uzbekiston Ssr, 1981.

Nizâmeddin en-Nisâbûrî. *eş-Şemsiyye fî'l-hisâb: Hesap Bilimlerinde Kılavuz*. Edited by Elif Baga. İstanbul: Türkiye Yazma Eserler Kurumu Başkanlığı, 2020.

Rashed, Roshdi. *Al-Khwārizmi: The Beginnings of Algebra.* London: SAQI, 2009.

Rosenfeld, Boris and Ekmeleddin İhsanoğlu. *Mathematicians, Astronomers, and Other Scholars of Islamic Civilization and their Works (7th-19th c.).* İstanbul: IRCICA, 2003.

Salih Zeki. *Âsâr-ı Bâkiye: Bilginlerin Yaşamları ve Yapıtları*. Vol. 3. Edited by Melek Dosay Gökdoğan, Remzi Demir, and Mutlu Kılıç. Ankara: Babil Yayınları, 2004.

———. *Kâmûs-i Riyâziyyât,* c.8. İstanbul Üniversitesi Library Rare Works Collection, TY00915.

Süveysî, Muhammed. "Hesap." In *TDV İslâm Ansiklopedisi*. accessed December 31, 2023. https://islamansiklopedisi.org.tr/hesap--matematik#1.

Umut, Hasan. "Theoretical Astronomy in the Early Modern Ottoman Empire: 'Alī al-Qūshjī's Al-Risāla al-Fatḥiyya." PhD diss., McGill University, 2019.