NAZARİYAT

Ulugh Beg's Treatise on the Altitude Circle*

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Abstract: Ulugh Beg was an influential figure in the history of astronomy and a key patron of the vibrant scholarly environment that flourished in ninth/fifteenth-century Samarkand. The intellectual milieu that emerged as a result of his patronage fostered dynamic exchanges between scholars and students across various scientific disciplines, leaving a lasting impact. Among his lesser-known works is a brief treatise on a problem in spherical astronomy. This treatise investigates the position of the altitude circle in various situations across different localities. Despite the simplicity of its subject matter, this text offers valuable insight into the nature of scholarly engagement under Ulugh Beg's rule. This paper presents a critical edition, English translation, and analytical study of the treatise, situating it within the broader intellectual context of Timurid Samarkand.

Keywords: Ulugh Beg, Samarkand, Qāḍīzāda, Shirwānī, Jaghmīnī, Jurjānī, spherical astronomy, altitude circle

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Introduction

Ulugh Beg (d. 853/1449), a Timurid ruler, patron of scientific activity, and accomplished scholar, left a profound legacy in the history of astronomy. His establishment of one of the most advanced medieval observatories, alongside his construction and support of educational institutions, transformed Samarkand into a thriving center of intellectual activity that attracted scholars from across the Islamic world. Among the most notable outcomes of these efforts was the *Zīj-i Sulṭānī* (also known as *Zīj-i Gurkānī* or *Zīj-i Ulugh Beg*), a Persian astronomical handbook often described as the most accurate set of pre-Keplerian astronomical tables.

The scholarly activities carried out under Ulugh Beg's patronage have been the subject of extensive research, supported by unique sources such as the letters of the Persian mathematician Ghiyāth al-Dīn Jamshīd al-Kāshānī (d. 832/1429, also known as "al-Kāshī") to his father and reports by the influential scholar Fatḥ Allāh al-Shirwānī (d. 891/1486).[1] These sources provide exceptional details about the intellectual dynamics of Samarkand that set it apart from other Islamic centers of learning, whose historical records are often less technical. They portray Samarkand as a vibrant scholarly environment where scientific discussions were regularly held at Ulugh Beg's *madrasa*, often featuring his active participation alongside other scholars. Many of these sources also attest to the continuation of such scholarly exchanges in informal settings, including gatherings in private homes.[2]

1 See E. S. Kennedy, "A Letter of Jamshid al-Kashi to His Father: Scientific Research and Personalities at a Fifteenth Century Court," *Orientalia* 29 (1960): 191–213; Aydın Sayılı, *Uluğ Bey ve Semerkanddeki İlim Faaliyeti Hakkında Gıyasüddin-i Kaşi'nin Mektubu* (*Ghiyāth al-Dīn al-Kāshī's Letter on Ulugh Bey and the Scientific Activity in Samarqand*) (Ankara: Türk Tarih Kurumu Basımevi, 1960); Abū al-Qāsim Qurbānī, *Kāshānī nāma: Aḥwāl wa āthār-i Ghiyāth al-Dīn Jamshīd Kāshānī* (Tehran: Markaz-i Nashr-i Dānish-gāhī, 1989); Mohammad Bagheri, *Az Samarqand bi Kāshān: Nāma-hā-yi Ghiyāth al-Dīn Jamshīd Kāshānī bi pidarash* (Tehran: Shirkat-i Intishārāt-i 'Ilmī va Farhangī, 1996); Mohammad Bagheri, "A Newly Found Letter of al-Kāshī on Scientific Life in Samarkand," *Historia Mathematica* 24 (1997): 241–56; George Saliba, "Reform of Ptolemaic Astronomy at the Court of Ulugh Beg," in *Studies in the History of the Exact Sciences in Honour of David Pingree*, edited by Charles Burnett, Jan P. Hogendijk, Kim Plofker, and Michio Yano (Leiden: E. J. Brill, 2004), 810-824; İhsan Fazlıoğlu, "Osmanlı felsefe-biliminin arkaplanı: Semerkand matematik-astronomi okulu," *Dîvân İlmî Araştırmalar,* 14, no.1 (2003): 1-66; İhsan Fazlıoğlu, "The Samarqand Mathematical-Astronomical School: A Basis for Ottoman Philosophy and Science," *Journal for the History of Arabic Science* 14 (2008): 3-68.

2 See, for example, Shirwānī's commentary on Ṭūsī's *Tadhkira*: MS Ahmet III 3314, f. 104b; MS Süleymaniye, Damat İbrahim Paşa 847, f. 109a, where he narrates a discussion that took place at Qāḍīzāda's house.

However, the availability of these historical details does not mean that our knowledge of Samarkand and its scholarly environment is complete. On the contrary, many questions about the nature and scope of these intellectual activities remain open, such that every document studied from this period contributes to expanding our understanding of these scholarly dynamics. In particular, examining Ulugh Beg's lesser-known works presents a valuable opportunity to deepen our understanding.

Among Ulugh Beg's writings is a short, unstudied treatise on the altitude circle, addressing a specific problem in spherical astronomy. This work is significant not only for the insights it provides into Ulugh Beg's early astronomical interests but also for what it reveals about his relationship with Qāḍīzāda al-Rūmī (d. after 844/1441).[3] Around 812/1409, Qāḍīzāda, one of the leading teachers of mathematical sciences at Ulugh Beg's *madrasa*, arrived in Samarkand. This was shortly after Ulugh Beg, who was still only fifteen years old, had been appointed the ruler of Transoxiana by his father Shāh-rukh (r. 807–851/1405–1447).[4] By 814/1411–2, Qāḍīzāda had completed a commentary on Jaghmīnī's *al-Mulakhkhaṣṣ fī al-hay'a*, which was likely a course-book for his lectures on theoretical astronomy. Ulugh Beg, who is said to have studied mathematical sciences under Qāḍīzāda, probably attended the latter's lectures at about this age,[5] and may have written his treatise on the altitude circle during this period. This early intellectual connection is further corroborated by the respect Ulugh Beg shows for Qāḍīzāda in the way he addresses his teacher at the end of the work.

Although there is no decisive evidence for the date of composition, Ulugh Beg's treatise on the altitude circle may have been written before Qāḍīzāda's commentary on the *Mulakhkhaṣṣ*. Several of the points discussed in Ulugh Beg's treatise are addressed in Qāḍīzāda's commentary, yet no textual dependence on the latter's work is evident in the former. A comparison of the two works suggests that Qāḍīzāda's com-

3 Regarding the discrepancy around Qāḍīzāda's death date, see Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 60.

4 See Beatrice F. Manz, "Ulugh Beg," in *The Encyclopaedia of Islam*, New Edition, edited by P. J. Bearman, Th. Bianquis, C. E. Bosworth. E. van Donzel, and W. P. Heinrichs (Leiden: Brill, 2000), 10: 812–4.

5 See Taşköprüzade, *al-Shaqā'iq al-nu'māniyya fī 'ulamā' al-dawla al-'Uthmāniyya* (Beirut: Dār al-Kutub al-'Arabī, 1975), 13; Kâtip Çelebi, *Sullam al-wuṣūl ilā ṭabaqāt al-fuḥūl*, ed. Ekmeleddin İhsanoğlu, Maḥmūd 'Abd al-Qādir al-Arnā'ūṭ, Ṣāliḥ Sa'dāwī Ṣāliḥ (Istanbul: IRCICA, 2010), 3:358; Manz, "Ulugh Beg," 813; Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 8.

mentary likely reflects Ulugh Beg's earlier discussions. This proposed dating implies that Ulugh Beg's treatise was composed during the early years of Qāḍīzāda's teaching in Samarkand, offering a glimpse into the young ruler's intellectual development under his master's guidance.

An anonymous contemporaneous commentary on Ulugh Beg's treatise offers further insights into the scholarly environment in Samarkand under Ulugh Beg's rule. Only one copy of this commentary has been identified, and in it the author's name is not given. The commentator tells us that he arrived in Samarkand to study the sciences, especially mathematics, with Ulugh Beg, but that he could not attend Ulugh Beg's teaching circle. He adds that he benefited from a treatise by Ulugh Beg, which he acquired through Qāḍīzāda: the same treatise we are studying in the present article.

This paper provides an edition, translation, and analysis of Ulugh Beg's treatise on the altitude circle. It situates the treatise within the intellectual context of Samarkand, examines its relationship with Qāḍīzāda's commentary on the *Mulakhkhaṣṣ*, and explores what it reveals about the dynamics of scientific activity under Ulugh Beg's patronage.

The Intellectual Environment at Ulugh Beg's madrasa in Samarkand and His Treatise on the Altitude Circle

None of Ulugh Beg's minor works are recorded in bio-bibliographical sources.[6] Nevertheless, his treatise on the altitude circle is cited by Fatḥ Allāh al-Shirwānī (d. 891/1486) in his commentary on Naṣīr al-Dīn al-Ṭūsī's (d. 672/1274) *al-Tadhkira fī ʿilm al-hayʾa*. Shirwānī was among the younger scholars who spent part of their early careers in Samarkand, pursuing studies in various sciences.[7] In his case, we know that

6 Among the minor works by Ulugh Beg was a treatise on calculating the sine of one degree, no copy of which has been found. Its contents are nevertheless known through Qūshjī's commentary on Ulugh Beg's *zīj*. For more on this treatise, see Fateme Savadi, "Recovering Two Lost Treatises on Approximating the Sine of 1° from Qūshjī's Commentary on *Zīj-i Sulṭānī*," *Nazariyat Journal for the History of Islamic Philosophy and Sciences* 11, no. 1 (2025): 165–213. Ulugh Beg also wrote a short treatise on parallax, which is under preparation for publication by the current author.

7 Apparently Shirwānī did not stay in Samarkand for a long time. All his available works belong to a later period of his life, and were composed in Shirvan and Ottoman lands. On 10 Muḥarram

he studied, among other subjects, Niẓām al-Dīn al-Nīsābūrī's (d. c. 730/1330) commentary on Ṭūsī's *Tadhkira* at Ulugh Beg's *madrasa* in Samarkand.[8]

In his own commentary on the *Tadhkira*, written in 879/1475,[9] Shirwānī offers detailed accounts of some events that occurred during his studies, including receiving a license from Qāḍīzāda[10] and an encounter with ʿAlī al-Qūshjī (d. 879/1474).[11] In the course of one such session, the discussion shifted to Ulugh Beg's treatise. Commenting on Ṭūsī's passage about the altitude circle, Shirwānī writes:[12]

وللسلطان المذكور هاهنا مقالة لا مزيد على تحقيقها وقد كتبتها[13] عند قرائتي شرح النظام وناولينها أوّلًا بإشارته الأستاذ رحمهما الله.

The aforementioned sultan has a treatise on this subject, whose accuracy is beyond surpassing. I had copied it during my [formal] reading sessions of Niẓām's commentary, and he initially handed it over to me at the master's suggestion—may God have mercy on them.

Based on Shirwānī's earlier references, the "sultan" and "master" mentioned in the passage refer to Ulugh Beg and Qāḍīzāda, respectively. The topic discussed preceding this sentence in al-Shirwānī's commentary aligns with the content of Ulugh Beg's treatise on the altitude circle. Thus, it is clear that Shirwānī is referring to the very treatise

857/1453, he wrote his commentary on *Āyat al-kursī* in Bursa and dedicated it to the ruler of Shirvan ʿAlāʾ al-Dīn Khalīl Allāh ibn Muʿīn al-Dīn Ibrāhīm (r. 821–867/1418–1463) (see MS Tehran, Majlis, Shūrā, 2882, pp. 407–499). On 3 Ramaḍān 879/1475, he finished his commentary on Ṭūsī's *Tadhkira*. On 4 Jumādá al-Thānī 883/1478, he dedicated his Persian translation of *al-Anwār li-aʿmāl al-abrār* (written by Jamāl al-Dīn Yūsuf ibn Ibrāhīm al-Ardabīlī, d. 799/1397) to the ruler of Shirvan Farrukh Yasār ibn Khalīl Allāh (r. 867–905/1463–1500) (see MS Tehran, Dānish-gāh, 2991). See also Scott G. Trigg, "From Samarqand to Istanbul: Astronomy and Scientific Education in the Commentaries of Fatḥallāh al-Shirwānī" (PhD diss., University of Wisconsin, 2016).

8 On the license he received for this study, see Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 43–4.

9 The commentator authorial colophon is reproduced in multiple copies. See MS Ahmet III, 3314, f. 368a; MS Damat İbrahim Paşa, 847, f. 212b:

قد فرغت من تسويده يد الشارح ليلة الأربعاء الثالثة من شهر رمضان عام تسع وسبعين وثمانمائة حامدًا لله ربّ العالمين ومصلّيًا على نبيّه محمّد وآله أجمعين.

10 See Fazlıoğlu, "The Samarqand Mathematical-Astronomical School."

11 Saliba, "Reform of Ptolemaic Astronomy at the Court of Ulugh Beg," 820.

12 MS Ahmet III, 3314, f. 98a; MS Süleymaniye, Damat İbrahim Paşa, 847, f. 84a.

13 In all copies I consulted, this word appears as *katabahā* with its subject going back to Ulugh Beg. Since this seems to be a corruption in the text, I changed it to *katabtuhā*.

examined in this paper. Given that this account shows Qāḍīzāda's role in initiating the dissemination of Ulugh Beg's work, we will explore its possible date.

The *qirā'a* ("formal reading") mentioned in Shirwānī's text refers to a common practice in medieval Islamic *madrasa*s, wherein a student would read a canonical text aloud in the presence of a master and perhaps other students and scholars. Upon completion, the student would receive a license certifying their study under that master (as was the case for Shirwānī). Other students, known as *sāmiʿūn* ("listeners" or "auditors"), might also receive a different kind of license for their participation.[14] Earlier in his commentary, Shirwānī refers to himself as one of the auditors attending reading sessions of Nīsābūrī's commentary (*wa-kuntu min al-sāmiʿīn li-sharḥ al-Niẓām*).[15] Following a discussion in one of these early sessions, Ulugh Beg replaced the original reciter with Shirwānī.[16] In the passage quoted above—also concerning the early parts of Nīsābūrī's commentary—Shirwānī refers to himself as the reciter (*ʿinda qirā'atī sharḥ al-Niẓām*), indicating that his transcription of Ulugh Beg's treatise likely took place shortly after this appointment. He noted that these sessions continued for approximately five years. He also includes in his commentary the license issued by Qāḍīzāda at the conclusion of his studies and just before his departure from Samarkand, dated 844/1440. From this information, we may infer that Shirwānī's study of Nīsābūrī's commentary spanned the years 839/1435 to 844/1440.[17]

It should be noted that the date of Shirwānī's license initially appears to contradict evidence from other sources. From Ulugh Beg's preface to his *zīj*, we know that Qāḍīzāda and another collaborator, Kāshī, passed away before the completion of the observations and the compilation of the *zīj*, which is commonly believed to

14 See George Makdisi, *The Rise of Colleges: Institutions of Learning in Islam and the West* (Edinburgh: Edinburgh University Press, 1981), 142–3; Adam Gacek, *Arabic Manuscripts, A Vademecum for Readers* (Leiden and Boston: Brill, 2009), 51–5; R. Sellheim, "Samā' (As a term in education)," in *Encyclopaedia of Islam, New Edition*, ed. P Bearman (Leiden: Brill, 1995), 8:1019–20; François Déroche, et al., *Islamic Codicology, an Introduction to the Study of Manuscripts in Arabic Script* (London: Al-Furqān Islamic Heritage Foundation, 2006), 332–4.

15 See Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 40.

16 Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 42; MS Ahmet III, 3314, f. 15b.

17 Apparently, Tāj al-Dīn al-Saʿīdī (fl. c. 844/1440) was another student who audited these particular reading sessions, since he copied a witness of Nīsābūrī's commentary on Ṭūsī's *Tadhkira* in the same year and same month in which Shirwānī received his license from Qāḍīzāda. See MS Tehran, Dānish-gāh, 8951, f. 214a.

have occurred around 841/1437.[18] However, Shirwānī's license is dated 844/1440 and was granted by Qāḍīzāda himself. Since Shirwānī reports that the license was issued shortly before his departure from Samarkand—an event he also reported to have taken place in the same year—the date of the license appears indisputable, allowing us to confidently infer that Qāḍīzāda was still alive in 844/1440. This undermines the earlier assumption of his death before 841/1437 and calls for further speculations about the *zīj*'s completion date (see below).

Shirwānī's claim to have studied Nīsābūrī's commentary in Samarkand between 839/1435 and 844/1440 is consistent with another of his scribal activities: in 843/1439, Shirwānī copied a manuscript of Ṭūsī's recension of Theodosius' *Sphaerica* in Samarkand. He says in the colophon of this witness:[19]

فرغ من انتساخه يوم الاثنين السادس والعشرين من ربيع الأوّل سنة 348 بمحروسة سمرقند الفقير إلى الله الغني فتح الله بن أبي يزيد بن عبد العزيز الشرواني عفى الله عنهم وعمّن قال آمين.

Transcription of [this copy] was completed on Monday, the 26th of Rabīʿ al-Awwal in the year 843/1439, in the protected city of Samarkand, by the one in need of God, the Self-Sufficient, Fatḥ Allāh ibn Abī Yazīd ibn ʿAbd al-ʿAzīz al-Shirwānī—may God pardon them and anyone who say Amen.

18 For a discussion of the issue regarding dating Ulugh Beg's *zīj* and Qāḍīzāda's death, see Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 60–1. In the introduction to the *Zīj-i Sulṭānī*, Ulugh Beg states that Qāḍīzāda passed away before the completion of the observations and the compilation of the *zīj* (MS Hyderabad, Salar Jung, 41, pp. 4–5):

«و در اثنای حال پیش از آنکه این مهم ساخته و پرداخته آید حضرت استادی شکّر الله مساعیه بجوار رحمت پروردگار پیوست.»

"And in the meantime, before this important matter was completed and perfected, my exalted master [Qāḍīzāda]—may God reward his efforts—passed into the vicinity of the mercy of the Lord."

Moreover, when Ulugh Beg first mentions Qāḍīzāda's name in the introduction, he employs the honorific Arabic phrase *ʿalayhi al-raḥma wa-l-ghufrān* ("may mercy and forgiveness be upon him"), indicating that Qāḍīzāda had already passed away at the time Ulugh Beg was writing this introduction. For an edition of the introduction, see L. P. E. A. Sédillot, *Prolégomènes des Tables Astronomiques d'Oloug-Beg* (Paris: Typographie de Firmin Didot frères, 1847), 285–91, reprinted in Fuat Sezgin, *Islamic Mathematics and Astronomy*, vol. 52.

19 MS Ankara, Milli, Tokat, 408, f. 177a.

Further evidence for Qāḍīzāda's teaching activities in 844/1441 at Ulugh Beg's *madrasa* comes from another student, Bakhshāyish ibn Bahāʾ al-Dīn al-Malaṭī, who also travelled to Samarkand for study. He copied several codices dated between 842–857/1438–1453. One of these codices includes a copy of Quṭb al-Dīn al-Shīrāzī's (d. 710/1311) *Nihāyat al-idrāk fī dirāyat al-aflāk*, dated end of 844/May 1441. The colophon reads as follows:[20]

تمّمت تحرير هذه النسخة اللطيفة بعون الله في قريب من شهر وهو موزع على ذي القعدة وذي الحجّة حجّة أربع وأربعين وثمانمائة في محروسة سمرقند في مدرسة واليه السلطان العظيم الشأن ألغ بيك بن أمير شاه رخ بن أمير تيمور كوركان مدّ الله على البرية ظلاله في زمان تدريس أستاذ الدنيا القاضي زاده الرومي طال بقائه. وأنا النحيف المحتاج إلى رحمة ربّه اللطيف بخشايش بن الشيخ بهاء الدين الملطي عنه عُفي رحم الله ناظرًا فيه يخرج الطيّبات من فيه.

I completed the transcription of this fine copy, with God's help, over the course of about a month, spanning the months of Dhū al-Qaʿda and Dhū al-Ḥijja in the year 844/1441, in the protected city of Samarkand, at the *madrasa* [founded by] its ruler, the great sultan Ulugh Beg ibn Amīr Shāh-rukh ibn Amīr Tīmūr Gūrkān—may God extend his shadow over creation—during the time of instruction of the master of the age, Qāḍīzāda al-Rūmī—may his life be prolonged. And I am the frail one in need of the mercy of his gracious Lord, Bakhshāyish ibn al-Shaykh Bahāʾ al-Dīn al-Malaṭī—who may be pardoned. May God have mercy on anyone who reads this and extract fine things from [what is] in it.

Shīrāzī's works were generally regarded as advanced texts in the *hayʾa* tradition. This suggests that Qāḍīzāda was not only alive in 844/1441 but also actively teaching advanced astronomy at that time.

Let us now return to the question of the composition date of Ulugh Beg's *zīj*. The base date for the astronomical tables in the *zīj* is the beginning of the year 841/1437, which provides a *terminus post quem* for its composition. The earliest known manuscript of the *zīj*—apparently Ulugh Beg's holograph copy—is dated 843/1439,[21] indicating that the *zīj* was complete by then or shortly before. However, as previ-

20 MS Tehran, Majlis, 6457, f. 91b.

21 MS Hyderabad, Salar Jung, 41.

ously noted, the *zīj*'s preface include a reference to Qāḍīzāda's death, which, based on Shirwānī's and Bakhshāyish's colophons, must have occurred after 844/1441. It is therefore possible that the 843/1439 manuscript represents an earlier, perhaps unfinished version of the *zīj*, composed before the preface was finalized.

As was mentioned above, at Qāḍīzāda's request, Shirwānī was granted access to Ulugh Beg's treatise and given the opportunity to transcribe it for himself during his study of Nīsābūrī's commentary on Ṭūsī's *Tadhkira*. Thus, the *terminus ante quem* from the composition of Ulugh Beg's treatise falls within the five-year period of Shirwānī's study of Nīsābūrī's commentary (839/1435 to 844/1440). Nevertheless, as our content analysis will later suggest, the actual composition of Ulugh Beg's treatise likely predates even this. Internal evidence indicates that it was written before Qāḍīzāda completed his commentary on Jaghmīnī's *Mulakhkhaṣṣ*, which dates to 814/1411–2. Before turning to the implications of this earlier date, we will first examine the extant copies of Ulugh Beg's treatise and the commentary it subsequently inspired.

Two manuscripts of Ulugh Beg's treatise on the altitude circle are known to be extant—one in Tehran and another in Dublin. Additional copies may have survived unnoticed, possibly overlooked by cataloguers due to the treatise's relatively short length. The two known witnesses are: MS Tehran, Kitābkhāna-yi Madrasa-yi Marwī, 877, ff. 64b–66a (hereafter MS M),[22] and MS Dublin, Chester Beatty Library, Arabic 3640, ff. 143a–144a (hereafter MS D). In addition to these two witnesses, a contemporaneous anonymous commentary on Ulugh Beg's treatise survives in a unique eleventh/seventeenth-century manuscript. This commentary provides full lemmata from the original text and thus serves as a third witness to Ulugh Beg's treatise: MS Kashan, Kitābkhāna-yi Mullā Muḥsin Fayḍ (Idāra-yi Farhang wa Irshād-i Islāmī), 27, pp. 141–171 (hereafter MS K).

In MS M, both in the table of contents at the beginning of the codex and at the end of the text, Ulugh Beg's treatise is referred to as *al-Risāla al-Malikiyya* ("the royal

22 See Riḍā Ustādī, *Fihrist-i nuskha-hā-yi khaṭṭī-yi kitābkhāna-yi Madrasa-yi Marwī-yi Tihrān* (Tehran: Kitābkhāna-yi Madrasa-yi Marwī, 1992), 323. Muṣṭafá Dirāyatī, *Fihristigān nuskha-hāyi khaṭṭī-yi Īrān* (*FANKHA*), vol. 19 (Tehran: Sāzimān-i Asnād wa Kitābkhāna-yi Millī-yi Jumhūrī-yi Islāmī-yi Īrān, 2013), 625.

treatise").[23] However, this title appears to be a later fabrication, or at least not one assigned by Ulugh Beg himself. Information from the aforementioned witnesses allows us to infer a more plausible title. Ulugh Beg's treatise in MS D begins with the following passage (written in a hand different than the scribe's):

> مقالة لطيفة شريفة في دائرة الارتفاع وكيفية دوران نقطة تقاطع الارتفاعية مع الأفق في كلّ البقاع مشتملة على كنوز التحقيق من نتائج طبع ملك العلماء وعالم الملوك، الملك الشهيد، ألغ بيك كوركان بن شاه رخ بهادر بن أمير تيمور رحمه الله.
>
> A subtle and dignified treatise on the altitude circle and the manner in which the point of intersection between the altitude circle and the horizon rotates in any locality, comprising precious verification produced by the disposition of the king of scholars and scholar of kings, the martyred king, Ulugh Beg Gūrkān ibn Shāh-rukh Bahādur ibn Amīr Taīmūr, may God have mercy on him.

From this introductory statement, the following title can be reasonably inferred: *Maqāla fī dāʾirat al-irtifāʿ wa-kayfiyyat dawarān nuqṭa taqāṭūʿ al-irtifāʿīyya maʿa al-ufuq fi kull al-biqāʿ* (a treatise on the altitude circle and the manner in which the point of intersection between the altitude circle and the horizon rotates in any locality).

The anonymous commentator, who provides little information about his own identity, uses the same phrase when referring to Ulugh Beg's treatise.[24] In the introduction to the commentary, the commentator writes:[25]

23 This title thus is reported in the catalogue of the Marwī library as the title of Ulugh Beg treatise. See Ustādī, *Fihrist-i nuskha-hā*, 323.

24 The author's name is not mentioned in the text. The scribe of MS K reproduced the colophon of his source copy without giving its date:

> وقد نقله من المسوّدة إلى البياض العبد المذنب الجاني سعيد بن عبد الرحيم بن شهاب الخواني حامد لله ومصلّيًا على نبيه.
>
> Since this source witness was copied from the draft copy to a fair copy, we can assume that it was produced in the process of the composition of the text and thus dated first half of the ninth/fifteenth century. This was probably the reason that the cataloger of the Kashan library assumed Saʿīd ibn ʿAbd al-Raḥīm was the author of the commentary. See Dirāyatī, *Fihristigān nuskha-hāyi khaṭṭī-yi Īrān*, 625.

25 MS K, pp. 141–43.

وبعد فإنّي منذ دخلت مدينة سمرقند ... طالبًا لتوفيق استفادة العلوم سيّما الرياضي ... من الحضرة ... السلطان ابن السلطان ابن السلطان ألغ بيك كوركان ابن شاه رخ بهادر بن أمير تيمور كوركان ... لمّا كان ذلك متعسّرًا عليّ ... كيف وإنّي ضارع وهو مالك رقاب السلاطين ... استعنت باستفادة تصنيف نشأ منه في دائرة الارتفاع وكيفية دوران نقطة تقاطع الارتفاعية مع الأفق في كلّ البقاع ... استطلعته من جناب الإمام التمام ... المشتهر فيما بين الأنام بقاضي زادة الرومي ... ثمّ جال في صدري ... أن أُقيّد كلّ ما وصل ذهني إليه ليصير بحيث كلّما نظرت فيه أعتبر عليه، لا ليكون شرحًا له كاشفًا عن احتمالاته، أو حاشية عليه مبيّنة لاعتباراته، لقصور طبيعتي عن درك مطالب أحكامه وفتور قريحتي من فهم مآرب أقسامه. والمرجو من كرم الناظر فيه بعين الإنصاف أن ينبّه على مواقع الخلل والاعتساف فإنّي بكثرة الحاجة متّصف وبقلّة البضاعة معترف.

Now, ever since I arrived in the city of Samarkand [...] I have sought the prosperity that lies in the pursuit of science, particularly mathematics [...] through the presence of [...] the sultan, son of the sultan, son of the sultan, Ulugh Beg Gurkan, son of Shāh-rukh Bahādur, son of Amīr Tīmūr Gūrkān [...] Since it proved complicated for me [to study under him, given...] how lowly I am, [standing] before him who holds absolute power over kings [...] I sought assistance by drawing benefit from a treatise authored by him on the altitude circle and the manner in which the point of intersection between the altitude circle and the horizon moves in different locations [...] I learned about it from the presence of the perfect Imam [...] known among the people as Qāḍīzāda al-Rūmī [...] Then it occurred to me [...] that I should record everything that has come to my mind [in the form of a book], so that whenever I study it, I may learn from it, not as a commentary examining its various meaning, nor as a gloss clarifying its intent, for my disposition falls short of grasping its conclusions, and my intellect is too feeble to comprehend its benefits. Yet I hope, through the generosity of the one who reads into it with a fair eye, that the places of error and aberration may be pointed out, for I am marked by deep need and openly acknowledge the scantness of my knowledge.

From this passage, it is clear that the commentator was not originally from Samarkand but traveled there to pursue knowledge, particularly of mathematics. He intended to study with Ulugh Beg but apparently was unable to do so, perhaps due to the fact that not everyone was permitted to attend Ulugh Beg's teachings, or that the level of instruction was too advanced for beginners. Shirwānī informs us that stu-

dents at Ulugh Beg's *madrasa* were ranked in three levels,[26] implying that there was a system in the school and that people could not attend classes randomly. Moreover, although we know that Ulugh Beg was involved closely in the *madrasa*'s affairs, no source mentions him as a teacher there—which does not mean that he did not have his private teaching circles.

The commentator says that instead of attending Ulugh Beg's lectures, he benefited from a treatise by Ulugh Beg, about which he learned from Qāḍīzāda. This account aligns with Shirwānī's narrative of Qāḍīzāda's role in the dissemination of Ulugh Beg's treatise. Later on, the commentator refers to his communication with Qāḍīzāda, implying that he possibly attended the latter's teaching sessions at Ulugh Beg's *madrasa* and discussed the contents of Ulugh Beg's treatise with him.[27] Other than these brief lines, no information is available about this commentator. We will discuss the commentator's approach and style below. But before that, we will analyze the contents of Ulugh Beg's treatise.

Analysis of the Text

Ulugh Beg's treatise on the altitude circle is a very short text with no divisions. It is about one particular problem in spherical astronomy discussed in a variety of *hay'a* works with varying details (including Nīsābūrī's commentary on Ṭūsī's *Tadhkira*, or commentaries written on Jaghmīnī's *Mulakhkhaṣ*). No diagram is included in the text. All the topics discussed relate to the altitude circle and its position relative to other circles on the celestial sphere. For the purposes of the present study, we can divide the text into different sections. In what follows, we will go through these sections and comment on each of them.

Before that, it is worth remembering that the altitude circle is a great circle on the celestial sphere that passes through the zenith and is thus perpendicular to the horizon. This circle is used to measure the position of celestial objects—planets, stars, the Sun, and the Moon—with respect to the horizon (see Fig. 1, in which *Zh* represents a quarter of the altitude circle). The altitude circle is part of a spherical coordinate system with two components: altitude and azimuth. The altitude is the

26 Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 45.

27 See MS K, p. 154.

arc along the altitude circle, measured between the horizon and the celestial object (*sh* in Fig. 1). The azimuth is the arc along the horizon, measured between the intersection of the altitude circle with the horizon and a reference point defined by another circle that cuts the horizon perpendicularly (*hE* in Fig. 1). This reference circle is called "the circle of initial azimuth"; it is drawn perpendicularly to the horizon at the east and west points and thus passes through the zenith.

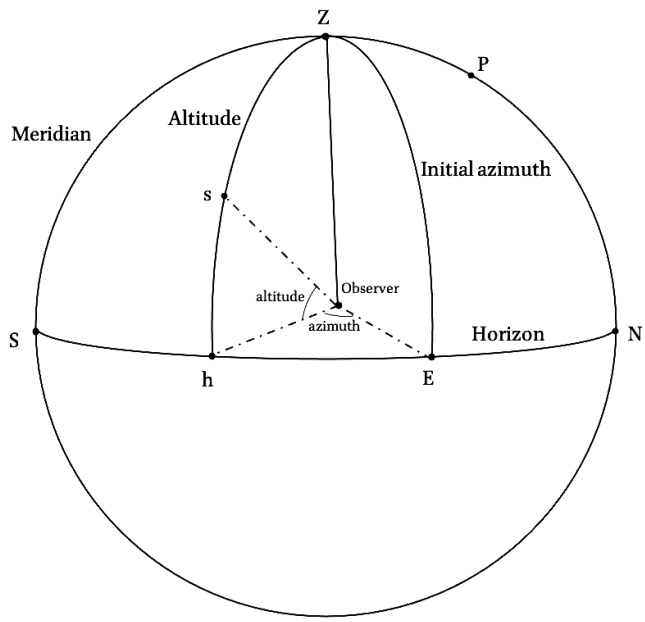

Fig. 1. Altitude-azimuth coordinate system.

The Altitude Circle and its Coincidence with the Local Meridian: A Response to Jaghmīnī (paragraphs 1–2)

Ulugh Beg begins the treatise with the following statement:

إعلم أنّ دائرة الارتفاع تنطبق على دائرة نصف النهار مرّتين في دورة كوكب، لا في يوم بليلته على ما قيل.

Be aware that the altitude circle coincides with the meridian twice during the revolution of each star, not during one nychthemeron, as it has been said.

In this passage, Ulugh Beg states that if we track the motion of a celestial object through its complete revolution around the Earth (approximately one nychthemeron), its altitude circle coincides with the meridian twice. As we will see below, Ulugh Beg explains that this statement is not universally true for all celestial objects at all terrestrial latitudes. Nevertheless, the crucial part of his argument is his refutation of the claim that this coincidence always occurs exactly twice within a single nychthemeron.

Although Ulugh Beg does not name these proponents directly, he is clearly referring to Jaghmīnī's *Mulakhkhaṣṣ* and his followers. Jaghmīnī states:

وهذه الدائرة تنطبق على دائرة نصف النهار في اليوم بليلته مرّتين.

This circle coincides with the meridian circle twice in a nychthemeron.[28]

Ulugh Beg refutes Jaghmīnī's claim by providing a counterexample. He explains that if the Moon is in conjunction with the Sun at the meridian, then by the time the Sun reaches the meridian for the second time, one nychthemeron will have passed. However, the Moon, due to its own proper motion, will not yet have reached the meridian again. Therefore, it is incorrect to assert that the altitude circles of all celestial objects coincide with the meridian twice in a nychthemeron.

Even if we disregard the proper motion of celestial objects, Jaghmīnī's statement is not universally valid. The coincidence of the altitude circle with the meridian depends on the declination of the stars and their daily motion relative to the horizon. Ulugh Beg states that if a star does not pass through the zenith, then for a permanently visible star, the coincidence occurs twice: once at its maximum altitude and once at its minimum altitude. If the star rises and sets, the second coincidence occurs at its maximum depression.

If the star passes through the zenith, the coincidence occurs only once at its minimum altitude (or maximum depression). The reason for this last case is that when a star passes through the zenith, its altitude circle coincides with the circle of initial

28 Sally P. Ragep, *Jaghmīnī's Mulakhkhaṣ: An Islamic Introduction to Ptolemaic Astronomy* (Cham: Springer, 2016), 110-111. The anonymous commentator, glossing Ulugh Beg's phrases *'alá mā qila*, says:

قال صاحب الجغميني وهذه الدائرة ينطبق على دائرة نصف النهار في اليوم بليلته مرّتين.

azimuth, not with the meridian. However, when the star is at the zenith, any great circle passing through the zenith—including the meridian—can serve as its altitude circle. Ulugh Beg argues nevertheless that if we follow the star's motion before it reaches and after it departs from the zenith, its altitude circle remains very close to the circle of initial azimuth. As an example, in Fig. 2, *ZnZ'* shows the altitude circle of a star before reaching the zenith, which is very close to *ZEZ'*. Thus, one cannot claim that the altitude circle coincides with the meridian when the star is at the zenith.

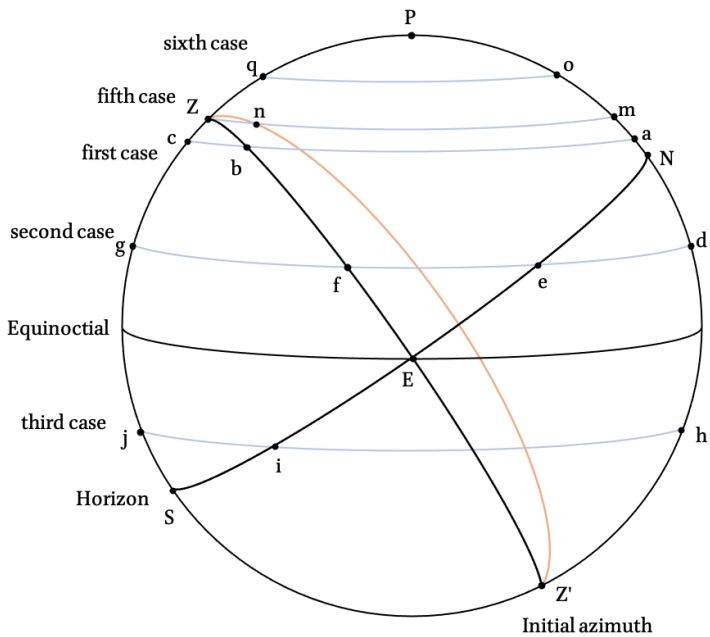

Fig. 2. The celestial sphere for latitudes above 45°.

Ulugh Beg further adds that if a star's circuit passes between the zenith and nadir, its altitude circle coincides twice with the circle of initial azimuth during a complete revolution (at point *f* in Fig. 2, for example). If a star passes through the zenith, its altitude circle coincides with the initial azimuth only once per revolution (circuit *mnZ* in Fig. 2). If a star's circuit passes between the zenith and the visible pole, or between the nadir and the invisible pole, its altitude circle never coincides with the initial azimuth (circuit *oq* in Fig. 2). These are the main topics discussed by Ulugh Beg in the first part of his treatise.

The Motion of the Azimuth Point along the Horizon (paragraphs 3–5)

In the next section of the treatise, Ulugh Beg first defines the azimuth point as the intersection of the altitude circle and the horizon. He then studies the motion of this point along the horizon at different localities. This section can be considered a gloss of Jaghmīnī's sentence:

> وتقطع دائرة الأفق على زوايا قائمة بنقطتين غير ثابتتين، بل منتقلتين على دائرة الأفق حسب انتقال الكوكب أو الشمس.
>
> It intersects the horizon circle at right angles at two points that are not fixed but rather shift along the horizon circle commensurate with the shifting of the star or the Sun.[29]

To analyze this motion, Ulugh Beg enumerates six cases based on the relationship between a star's circuit and the initial azimuth:

1) The circuit intersects the initial azimuth but not the horizon (circuit *abc* in Fig. 2).
2) The circuit intersects both the initial azimuth and the horizon (circuit *defg* in Fig. 2 and Fig. 3).
3) The circuit does not intersect the initial azimuth above the horizon but does intersect the horizon (circuit *hij* in Fig. 2).
4) The circuit is tangent to the initial azimuth and intersects the horizon (circuit *klZ* in Fig. 3).
5) The circuit is tangent to the initial azimuth but does not intersect the horizon (circuit *mnZ* in Fig. 2).
6) The circuit does not reach the initial azimuth, regardless of whether it intersects the horizon (circuits *oq* and *rst* in Fig. 2 and Fig. 3).

In the first case, the circuit (*abc* in Fig. 2) does not intersect the horizon, meaning the star is permanently visible. Since it does intersect the initial azimuth, the azimuth point traverses the entire horizon in a full revolution. If the star is at point *a*, its azimuth is at *N*. When it reaches *b*, the azimuth is at *E*; when the star is at *c*, the azimuth is at *S*. Thus, the azimuth completes a full revolution along the horizon.

29 Sally P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, 110–11.

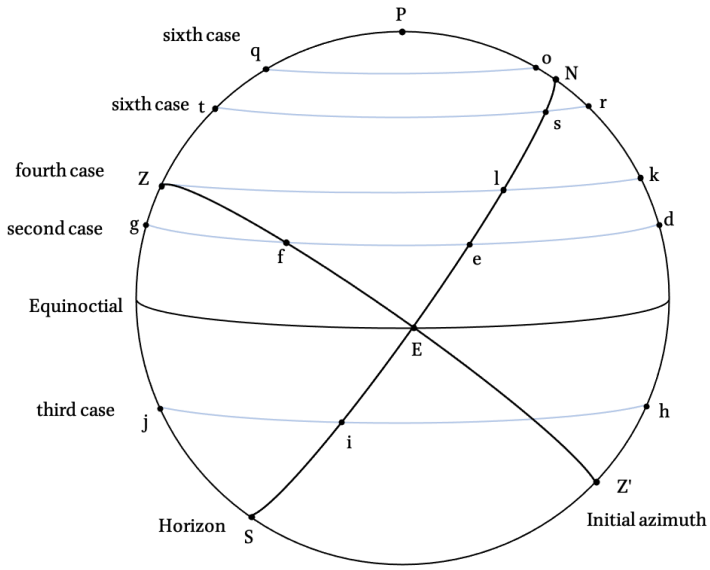

Fig. 3. The celestial sphere for latitudes below 45°.

In the second case, the circuit (*defg* in Fig. 2) intersects both the horizon and the initial azimuth. The star rises from point *e* on the horizon, marking the beginning of the azimuth point's motion along the horizon. When the star reaches *f*, the azimuth point is at *E*, with the distance between *e* and *E* equal to the star's ortive amplitude. When the star reaches *g*, the azimuth point reaches *S*, meaning the distance traveled from *e* to *S* equals the ortive amplitude plus a quarter-circle. This sequence occurs during the first half of the star's circuit. By analogy, in the second half of the circuit, the azimuth point moves along an arc equal to the occasive amplitude plus a quarter-circle. Thus, over a full revolution of the star, the azimuth point traverses an arc equal to half a circle plus the ortive and occasive amplitudes.

In the third case, the circuit (*hij* in Fig. 2) does not intersect the initial azimuth above the horizon but does intersect the horizon at *i*, marking the star's rising point. The segment *Ei* represents the star's ortive amplitude. When the star reaches *j*, the azimuth point is at *S*, meaning it has traversed an arc equal to half a circle minus the ortive amplitude. Thus, over a full revolution, the azimuth point moves along an arc equal to half a circle minus the sum of the ortive and occasive amplitudes. If the star lies on the equinoctial, its ortive and occasive amplitudes are zero, and in localities

other than the equator, the azimuth point moves exactly half a circle along the horizon. It should be noted that when Ulugh Beg states that the circuit does not intersect the initial azimuth, he means that it does not intersect the portion of the azimuth above the horizon.

In the fourth case, the circuit (*klZ* in Fig. 3) intersects the horizon but is merely tangent to the initial azimuth at the zenith. The star rises at *l*, with its ortive amplitude given by *lE*. When it reaches *Z*, its azimuth point is at *E*, meaning it has traversed an arc equal to the ortive amplitude. The same occurs on the other half of the circuit, so in total, the azimuth point moves along two arcs equal to the ortive and occasive amplitudes.

In the fifth case, the circuit (*mnZ* in Fig. 2) does not intersect the horizon but is tangent to the initial azimuth at *Z*. When the star is at *m*, its azimuth point is at *N*; when it reaches the zenith, its azimuth is at *E*, meaning it has traversed one-quarter of a circle. Thus, over a full revolution, the azimuth point moves along half of the horizon.

Ulugh Beg's final case includes two subcases: (1) circuits that do not intersect either the horizon or the initial azimuth (circuit *op* in Fig. 2 and Fig. 3) and (2) circuits that do not intersect the initial azimuth but do intersect the horizon (circuit *rst* in Fig. 3).

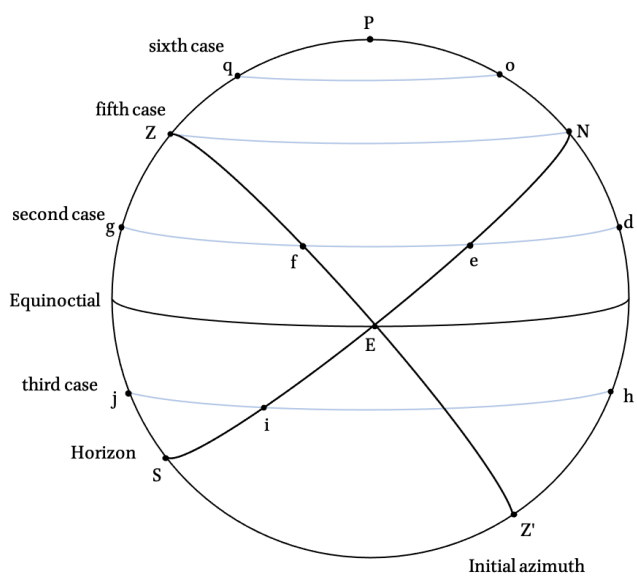

Fig. 4. The celestial sphere for latitude of 45°.

In the first subcase, when the star is at *o*, the azimuth point is at *N*, and when it reaches *q*, the azimuth is once again at *N*. However, in between, the azimuth undergoes an oscillatory motion along the horizon. In Fig. 5, the circuit, horizon, and meridian are presented from a different perspective. As the star moves from *o*, the angle between the altitude circle and the meridian increases until it reaches *u*, where it stops increasing and begins to decrease. The point *u* marks the place where the altitude circle becomes tangent to the circuit, meaning angle *ZuP* is 90°. If we calculate the maximum angle *vZN* using the sine theorem, we will have $\frac{\sin uP}{\sin ZP}$, where arc *ZP* is the complement of the locality's latitude, and arc *uP* is the complement of the star's declination. This relationship, which is presented by Ulugh Beg, allows us to determine arc *vN*, representing the azimuth point's oscillation range during half of the circuit. As the star moves from *o*, the azimuth point shifts until it reaches *v*, then moves back toward *N* and beyond to *v'*, before returning toward *v*. The total oscillation spans an arc equal to twice *vN*.

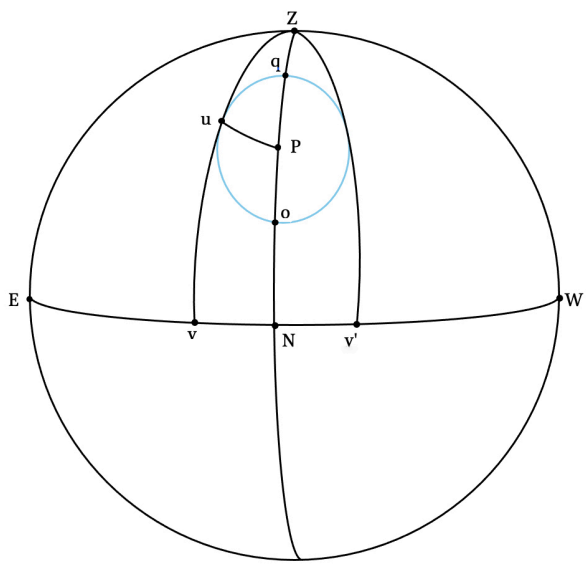

Fig. 5. The circuit for Ulugh Beg's sixth case.

In the second subcase, where circuits intersect the horizon (Fig. 6), this oscillation is interrupted at the rising and setting points of the star. Depending on the declination and latitude, the azimuth point starts its motion from *S*, located between *N* and *v*, and moves until it reaches *v*. It then reverses direction, passing *N* and reaching *v'*, before moving back toward *N*, stopping at *S'* before reaching it.

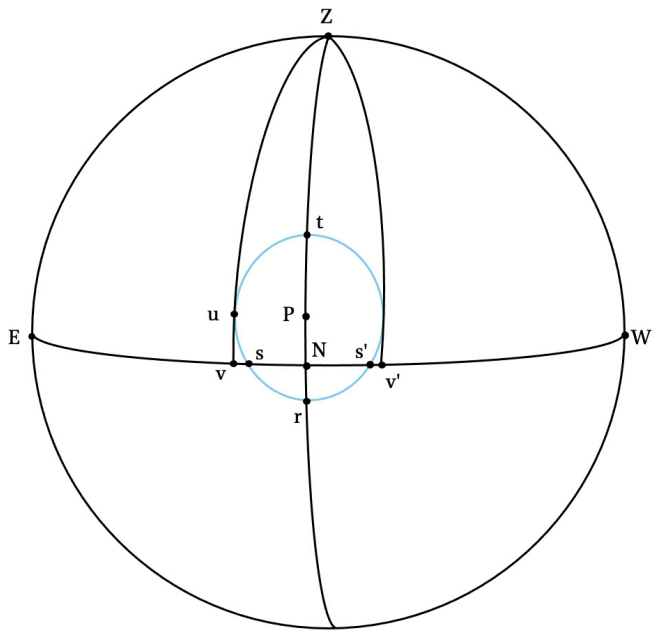

Fig. 6. The circuit for Ulugh Beg's sixth case, intersecting the horizon.

Ulugh Beg briefly acknowledges a possible objection regarding the azimuth motion in cases one, five, and six, noting that in these cases, the circuits may be tangent to the horizon, which could necessitate excluding the tangency point from the azimuth's path. If so, the azimuth's trajectory should be described as a series of distinct arcs. Although Ulugh Beg provides instructions for such cases, the textual transmission is unclear, and manuscript variations prevent a definitive resolution.

The Correspondence between the Cases Discussed in the Previous Section and Different Localities (paragraphs 6–8)

In the next section of the treatise, Ulugh Beg states that not all the cases discussed previously apply to all latitudes. At the equator, the initial azimuth coincides with the equinoctial. As a result, stars that are not on the equinoctial intersect the horizon but do not cut the initial azimuth. This situation corresponds to Ulugh Beg's third case in the previous section.

If we assume that a star has no proper motion, it rises at one point on the horizon and sets at another. In this case, the arc along which the azimuth point moves on

the horizon equals half a circle minus the ortive and occasive amplitudes, which are equal. The rising and setting points of stars on the equinoctial are the east and west points on the horizon. Therefore, the azimuth point is either at the east or the west.

If, however, we assume that stars have their own proper motions, their azimuth points move along different arcs as they transition from one circuit to another. In general, if a star rises and sets on the same side of the equinoctial, the arc along which its azimuth moves on the horizon is given by half a circle minus the ortive and occasive amplitudes, which are not equal. However, if the star rises on one side of the equinoctial and sets on the other, multiple scenarios arise, some of which Ulugh Beg discusses.

First, he confirms that at the equator, his statement in the third case—that the azimuth point moves along an arc smaller than half a circle—remains valid. He then examines the case where a star crosses the equinoctial at either its rising or setting. If a star crosses the equinoctial at rising (Fig. 7), its ortive amplitude is zero. However, as the star moves to a new circuit after rising, its azimuth point moves along an arc equal to half a circle minus the occasive amplitude. Conversely, if the star crosses the equinoctial at setting, the occasive amplitude is zero, and the arc along which the azimuth moves is half a circle minus the ortive amplitude. Although Ulugh Beg does discuss this, we should keep in mind that the case of a star crossing the equinoctial, the altitude circle may coincide with the meridian.

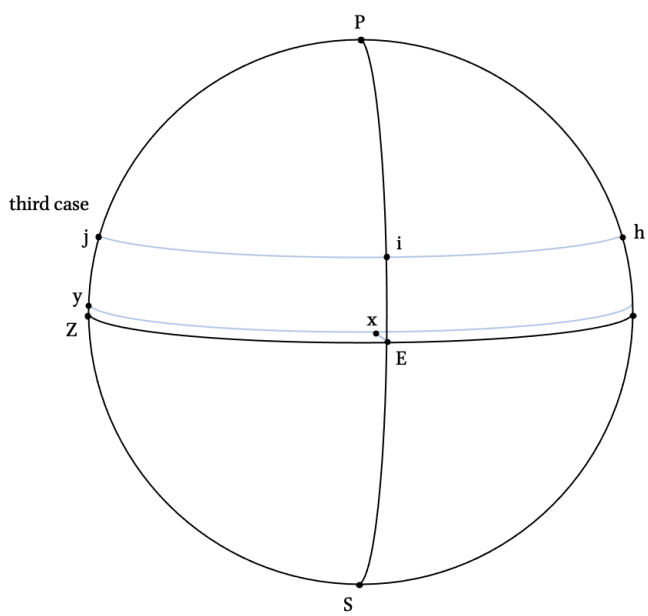

Fig. 7. The celestial sphere for localities at equator.

Ulugh Beg's next case considers a star crossing the equinoctial at the zenith. In this case, the arc along which the azimuth point moves equals the sum of the ortive and occasive amplitudes. If the star crosses the eastern side of the equinoctial (the region between the zenith and the east), the arc equals half a circle plus the ortive amplitude minus the occasive amplitude. If the star crosses the equinoctial on its western side, the arc equals half a circle plus the occasive amplitude minus the ortive amplitude. These are the only cases Ulugh Beg discusses for the equator.

For oblique horizons up to a latitude of 45° (as shown in Fig. 3), Ulugh Beg states that all cases from the previous section apply except the first and fifth cases. At 45° latitude (Fig. 4), all cases apply except the first and fourth. From 45° up to 90° latitude (Fig. 2), all cases apply except the fourth. At 90° latitude, Ulugh Beg briefly notes that the only possible motion is the full revolution from the first case. However, since there is no initial azimuth, there is no intersection condition. Even if stars move from one circuit to another, their azimuth point still completes a full revolution along the horizon. Therefore, he does not discuss any additional cases.

In the final part of the treatise, Ulugh Beg returns to the topic of proper motion and explains that this motion causes stars to change circuits. He states that for stars whose circuits correspond to the first case, nothing changes. The cases that consider the ortive and occasive amplitudes remain unaffected, but in the fifth and sixth cases, slight, almost imperceptible changes occur. Apparently, Ulugh Beg assumes that any changes in circuits still fall within the same case classifications. However, if a star changes its circuit such that it transitions from one case to another, the arc along which the azimuth point moves is affected.

In the fifth case, where a circuit is tangent to the initial azimuth, any change in the circuit alters the case under which it falls. In the sixth case, changes in the circuit affect the radius of the circuit and, consequently, the position of point ν in Fig. 5. However, as Ulugh Beg states, these changes are imperceptible.

Al-Jurjānī and Qāḍīzāda on al-Jaghmīnī's Passage

Now that we have fully analyzed the contents of Ulugh Beg's treatise, it is worth examining how the commentaries on Jaghmīnī's *Mulakhkhaṣṣ* that were available to Ulugh Beg elaborated on this topic. In his commentary, al-Sayyid al-Sharīf al-Jurjānī (d. 816/1413) offers two brief remarks on Jaghmīnī's sentence quoted above. He first states:

ودائرة الارتفاع تنطبق على دائرة نصف النهار في يوم بليلته مرّتين إحديهما عند وصول الكوكب إلى دائرة نصف النهار فوق الأفق والأخرى عند وصوله إليها تحت الأفق.

The altitude **circle coincides with the meridian circle twice in a nychthemeron**: once when the star reaches the meridian above the horizon, and again when it reaches [the meridian] below the horizon.[30]

As we saw above, this statement works only for the stars that rise and set, do not have any proper motion, and do not pass through the zenith. So, Jurjānī did not have the concerns Ulugh Beg had, and his statement only works for certain stars. He then says:

واعلم أنّ الكوكب ما دام على معدّل النهار ففي خطّ الاستواء دائرة ارتفاعه هي المعدّل فلا تنطبق دائرة ارتفاعه على نصف النهار أصلًا، ولا تكون نقطتا تقاطعها مع الأفق منتقلتين بل ثابتتين لأنّهما نقطتا المشرق والمغرب بعينهما.

Be aware that as long as the star is on the equinoctial, at the equator, its altitude circle is identical to the equinoctial itself. Thus, the altitude circle [of the star] never coincides with the meridian. [Furthermore], the two points where [the altitude circle] intersects the horizon do not shift [along the horizon]; rather, they remain fixed, as they are the points of the east and west.[31]

Jurjānī here considers only one case in which stars at the equinoctial are observed at the equator. In this case the altitude circle coincides with the equinoctial, and it never coincides with the meridian. As the altitude circle of these stars remains

30 MS British Library, ADD 23397, f. 15b; MS Süleymaniye, Kara Çelebi-Zade Hüsamettin, 348, f. 129b.

31 MS British Library, ADD 23397, f. 15b; MS Süleymaniye, Kara Çelebi-Zade Hüsamettin, 348, f. 129b.

the same equinoctial, their azimuth is either the east point or the west point on the horizon. These are the only comments Jurjānī makes about Jaghmīnī's sentence. Qāḍīzāda's commentary is a little more extensive.

وهذه الدائرة أي دائرة ارتفاع كلّ نقطة إذا لم تكن تلك النقطة ثابتة أو مارّة بسمت الرأس أو القدم **تنطبق على دائرة نصف النهار في اليوم بليلته** على ما اصطلح عليه الحُسّاب **مرّتين** مرّة عند وصولها إلى التقاطع الأعلى بين مدارها ودائرة نصف نهارها ومرّة عند وصولها إلى التقاطع الأسفل، لا أنّ إحديهما عند وصولها إلى دائرة نصف النهار فوق الأفق والأخرى عند وصولها إليها تحت الأفق إذ لا يستقيم فيما لا يغرب وكذا فيما لا يطلع. وأمّا إذا كانت النقطة ثابتة كالقطبين فدائرة ارتفاعها منطبقة على دائرة نصف النهار دائمًا. وأمّا إذا كانت مارّة بسمت الرأس أو القدم، ففي خطّ الاستواء لا انطباق أصلًا. (حقيقةً وحسًّا إذا لم تكن النقطة متحرّكة بغير الحركة الأولى وحسًّا إذا كانت متحرّكة بغيرها أيضًا لما عرفت من أنّ نقطة السمت لا تنتقل حينئذ حسًّا) وأمّا في غيره فتنطبق عليها في اليوم بليلته مرّة لا مرّتين.

This circle, i.e., the altitude circle of every point [on the celestial sphere], if that point is not fixed, or passing through the zenith or nadir, **coincides with the meridian circle twice in a nychthemeron** as it is conventionally understood by those who practice calculations. One [of these coincidences] is when that [point] reaches the upper intersection between its circuit and the meridian, and another when it reaches the lower intersection. It is not [as has been said by some people] that one of them is when [the point] reaches the meridian above the horizon, and another is when it reaches it below the horizon. Because this [statement] does not fit with the case [of stars] that do not set, [i.e., permanently visible stars], and those that do not rise [i.e., permanently invisible stars]. If that point is fixed, like the two poles [of the celestial sphere], their altitude circle always coincides with the meridian circle. If [that point] passes through the zenith or nadir, then at the equator, there will be no coincidence [between the altitude circle and the meridian] at all (neither factually nor sensationally if that point has no motion other than the universal motion, but only sensationally if it has another motion as well. This is because of what has been [mentioned previously and] known: that the azimuth point does not move sensationally). Nevertheless, in other [localities, the altitude circle] coincides [the meridian] once in a nychthemeron, not twice.[32]

32 MS Süleymaniye, Ayasofya, 2662, f. 22b; MS University of Pennsylvania, Lawrence J. Schoenberg, 408, f. 21a.

From the style of this passage, it seems that Qāḍīzāda is both reflecting on Ulugh Beg's treatise and correcting Jurjānī. He first excludes from his discussion the fixed points on the celestial sphere, i.e., the poles, as well as the zenith and nadir, to avoid the kind of complexities addressed by Ulugh Beg in his treatise. He then adds the phrase "as it is conventionally understood by those who practice calculations" to emphasize that this is not the view he prefers, and that it is only common among astronomers. Next, by a slight rephrasing, he avoids the issue in Jurjānī's text regarding permanently visible stars. Instead of saying that stars reach the meridian above and below the horizon, he uses the phrases "the higher intersection" and "the lower intersection." He clearly does not hesitate to show his disagreement with Jurjānī.

In MS Ayasofya 2662, which was, according to its colophon, copied in 820/1417 by Qāḍīzāda himself, a marginal note was added to this passage by the scribe. This marginal note, which is included inside parentheses in the text quoted above, shows another link between Ulugh Beg's treatise and Qāḍīzāda's commentary. The main topic of this note is how the stars' proper motion affects the behavior of the altitude circle. Qāḍīzāda does not follow Ulugh Beg's instruction for distinguishing different cases exactly. Rather, he says here that for stars that pass through the zenith at the equator, if we assume that they do not have any proper motion, there will be no coincidence between their altitude circle and the meridian. But if we assume that they have proper motions, he notes without providing much detail, the change in behavior of the altitude circle will not be perceivable. As we saw in Ulugh Beg's discussion, if the stars cross the equinoctial, their altitude circles might coincide with the meridian.

I assume that all these comments and modifications are posterior to Ulugh Beg's treatise, as one may question the purpose of Ulugh Beg's text if the main idea had already been presented in Qāḍīzāda's commentary, which was itself dedicated to Ulugh Beg. One might also ask why, if Ulugh Beg's treatise were written after Qāḍīzāda's commentary, the former does not rely on the latter textually. It is plausible that Qāḍīzāda refrained from incorporating Ulugh Beg's discussions into his commentary so as not to undermine his student's work. Since Qāḍīzāda's commentary was completed in 814/1411–2, we may reasonably speculate that Ulugh Beg's treatise was composed sometime before this date.

Until this point, we have examined two commentaries written in close relation to the intellectual circles to which Ulugh Beg belonged. But the passage from Jaghmīnī's

Mulakhkhaṣṣ was elaborated and commented upon by other scholars as well.[33] One interesting case is Kamāl al-Tūrkānī (fl. 760/1359), whose work covers several topics discussed by Ulugh Beg.

Little is known about the career of this commentator on Jaghmīnī's *Mulakhkhaṣṣ*. He is usually cited in modern scholarship under the name Kamāl al-Dīn al-Turkamānī. But he himself always consistently wrote his name in manuscripts as Kamāl al-Tūrkānī. In addition to this Shirwānī cites his name as Tūrkānī.[34] A holograph witness of Tūrkānī's commentary is extant in MS Princeton University Library, Garrett, 505H, which was copied in 755/1354 in Gülistan and (dedicated to Maḥmūd Ṣāḥib al-Dīwān). Tūrkānī's ownership notes are also found in the two following manuscripts, signed respectively in 747/1346 in New Sarai and in 757/1356 in Gülistan: MS Bodleian Library, Thurston, 3; MS Leiden, Or. 110. In the former manuscript, Tūrkānī scribed two short texts (ff. 147b–148a) in 760/1359 and 763/1362. Different witnesses of his commentary are dedicated to three different individuals: Jalāl al-Dīn Maḥmūd Jānī Beg Khān (r. 742–759/1342–1358), al-Amīr Ramaḍān (apparently the son of the former), and Kamāl al-Dīn Amīr Maḥmūd Ṣāḥib al-Dīwān (apparently the vizier of Jānī Beg).

In his commentary on Jaghmīnī's *Mulakhkhaṣṣ*, commenting on the sentence in which Ulugh Beg was interested, Tūrkānī says:

ودائرة ارتفاع كلّ كوكب مداره اليومي غير مارّ بسمت الرأس تنطبق على دائرة نصف النهار في اليوم بليلته مرّتين. وذلك الانطباق إنّما يكون عند وصول الكوكب إلى كلّ من تقاطعي مداره مع دائرة نصف النهار إذ هي لمرورها بأقطابها تنصف جميع المدارات اليومية. وإنّما قيّدنا مدار الكوكب بعدم مروره بسمت الرأس لأنّه لو مرّ به لم يلزم انطباق دائرة ارتفاعه على دائرة نصف النهار في اليوم بليلته مرّتين، إذ مداره على هذا التقدير إمّا معدّل النهار أو غيره. فإن كان الأوّل كان دائرة ارتفاعه في تلك الدورة هي معدّل النهار حسًّا فلا يلزم

33 It should be noted that the same problem regarding the position of the altitude circle is discussed in other texts and commentary traditions as well. In his *Tadhkira*, Ṭūsī briefly introduces the altitude circle, mentioning the main cases in which it may coincide with the meridian or the initial azimuth. However, he avoids problematic assertions such as those found in Jaghmīnī's *al-Mulakhkhaṣṣ*, or the detailed discussions later developed by Ulugh Beg. Quṭb al-Dīn al-Shīrāzī, drawing on Ṭūsī and other earlier authorities, focused on different aspects of the altitude circle. In the earlier versions of his *Nihāyat al-idrāk* he repeated Jaghmīnī's problematic assertion, though he later revised the text, indicating that he no longer agreed with Jaghmīnī.

34 MS Ahmet III, 3314, f. 250a; MS Süleymaniye, Damat İbrahim Paşa, 847, f. 132a.

الانطباق لا مرّتين ولا مرّة واحدة. وإن كان غيره يلزم الانطباق مرّة واحدة فقط وذلك عند وصول الكوكب إلى التقاطع الأسفل لمداره مع نصف النهار. وأمّا عند وصوله إلى التقاطع الأعلى فتنطبق دائرة ارتفاعه بحسب حركتها تلك على دائرة المشرق والمغرب لا على دائرة نصف النهار.

The altitude circle of every star whose day circuit does not pass through the zenith coincides with the meridian twice in a nychthemeron. This coincidence occurs when the star reaches any of the two intersections of its circuit with the meridian. This is because the meridian, due to its passing through the poles of the circuits, halves all of them. The reason we specified that the circuit does not pass through the zenith is that, if it passes through it, it will not follow that the altitude circle coincides with the meridian twice in a nychthemeron. This is because, in that case, its circuit is either equinoctial, or something other than that. If the former is the case, its altitude circle in that revolution is sensationally the equinoctial, so no coincidence follows, neither twice, nor even once. If [the circuit] is something else, only one coincidence follows which is when the star reaches the lower intersection of its circuit with the meridian. At its reaching the upper intersection, due to its same [universal] motion, the altitude circle coincides with the east-west circle, [i.e., the initial azimuth], not the meridian.[35]

First, by adding some explanatory phrases ("every star whose day circuit does not pass through the zenith") to Jaghmīnī's sentence, Tūrkānī avoid the issue addressed by Ulugh Beg and Qāḍīzāda. He further explains that he did so because if a star passes through the zenith, there will be two possibilities: either the circuit is the equinoctial, or it is not. In the first case (which was referred to by Jurjānī and Qāḍīzāda as well) the altitude circle is the equinoctial, and thus there will be no coincidence between the altitude and the meridian. Like Qāḍīzāda in his marginal note, Tūrkānī uses the word "*ḥissan*" probably to exclude the case of stars with their own proper motions. In the second case in which a non-equinoctial circuit passes through the zenith, like Ulugh Beg, Tūrkānī says that the altitude coincides once with the meridian, but at the zenith it coincides with the initial azimuth. Contrary to Jurjānī, but like Qāḍīzāda, Tūrkānī speaks about the intersection of the circuits with the meridian to include both stars that set and rise and the permanently visible stars. We see that all these cases were addressed by Ulugh Beg and Qāḍīzāda but using relatively different terminologies and approaches.

35 MS Princeton University Library, Garrett, 505H, ff. 17b–18a; MS Süleymaniye, Ayasofya, 2653, f. 102a; MS Columbia University Library, Smith-Plimpton, Or 148, p. 25.

So, it is clear that, in this particular passage, Jurjānī was not using Tūrkānī's commentary. The case of Qāḍīzāda, however, is unclear. His use of the term *ḥissan* in the aforementioned marginal note may imply Tūrkānī's influence on him. But there is no decisive evidence in the main text indicating that Qāḍīzāda was using Tūrkānī's commentary. On the other hand, Ulugh Beg's terminology and method is so distinctive that makes it hard to imagine any influence from Tūrkānī.

An English Translation of Ulugh Beg's Treatise on the Altitude Circle

[1] Be aware that the altitude circle coincides with the meridian twice during the revolution of a star, not in a single nychthemeron, as some have claimed. This is because if we assume that the Moon is in conjunction with the Sun when [the Sun] first reaches [the meridian], [then by the time the Sun reaches the meridian] for the second time, one nychthemeron will have passed, but the Moon will not yet have reached [the meridian, lagging behind] by the amount it has moved [due to its own motion and] went ahead [of the Sun in sequence of the zodiacal signs] during that period. If a star does not pass through the zenith, for the stars that rise and set, one [coincidence occurs] above the horizon and the other below it, but for the permanently visible stars, [these two coincidences occur] at their highest and lowest altitudes. However, if [the star] does pass [the zenith, the coincidence occurs only] once—at its lowest altitude, or at its greatest depression [below the horizon]. At its highest [altitude, however, the altitude circle coincides] with [the circle of] the initial azimuth. This is because, just before [the star] reaches [the zenith, the altitude circle] is nearly coinciding with the initial azimuth. Nevertheless, it is permissible to say that [this coincidence occurs] with the meridian, because at that moment, the circle passing through the zenith is an altitude circle. However, the altitude circle separates from the initial azimuth once the star passes the zenith. So how can one say that it coincided with the meridian before this separation?

[2] [The altitude circle] also [coincides] with the initial azimuth at two nearly equal altitudes or two nearly equal depressions, provided that the star passes [somewhere] from the zenith toward the hidden pole until its circuit becomes tangent to the nadir. If the star passes through the nadir, [the altitude circle coincides with the initial azimuth only] once. However, if the star passes [somewhere] from the zenith toward the visible [pole]—meaning [the pole] that is toward the latitude of the locality—or [from the nadir] toward the hidden [pole], which is the opposite, the altitude circle does not coincide with the initial azimuth.

[3] The point where the altitude circle intersects the horizon is called the azimuth point. We explain this because the azimuth [arc] ceases to exist twice in a revolution, [while the azimuth point] moves along the horizon depending on the position of the circuits [of the stars]. A circuit may intersect either the initial azimuth alone, or both [the initial azimuth and] the horizon, or it may not intersect the initial azimuth and [lie] toward the hidden [pole] from [the initial azimuth], or, it may be tangent to [the initial azimuth] and intersect the horizon, or it may be tangent to [the initial azimuth] without intersecting [the horizon], or it may not touch [the initial azimuth at all], whether it intersects the horizon or not. The circuits in the last three cases are toward the visible [pole], making [a total of] six divisions.

[4] In the first case, [the azimuth point] completes a [full] revolution [along the horizon]. In the second case, it moves along an arc greater than half [a circle] by an amount equal to the sum of the star's ortive and occasive amplitudes. In the third case, [it moves] along [an arc equal to] half [a circle] or smaller than that by an amount equal to the sum [of these amplitudes]. In the fourth case, [the azimuth point moves] along two arcs, each of which is smaller than a quarter-circle by an amount equal to the complement [of the ortive and occasive amplitudes]. In the fifth case, [it moves] along [an arc equal to] half [a circle]. In the sixth case, if [the circuit] does not intersect the horizon, [the azimuth point] begins [its motion] from the west and oscillates [along an arc] smaller than half [a circle], with half of its magnitude determined by [the size of] the circuit. The sine of this [half-arc] is obtained by dividing the sine of the complement of the [star's] declination by the sine of the complement of the locality's latitude (with a reduction of one sexagesimal degree). However, if [the circuit] does intersect [the horizon], this is not the case; rather, [the oscillation] stops near the midpoint of that arc.

[5] The star does not move from the quadrant of the altitude circle in which it was to the other. One may say that, in the first case, [the azimuth point] does not complete its revolution, nor does it move as we described in the fifth and sixth cases with respect to a point associated with the visible pole—[this point] being the midpoint of its path in [these] two last cases—because the altitude may cease to exist. If this is correct, then [the azimuth point] moves along the eastern quarter as well as western and southern [quarters], and also along two arcs in the fifth and sixth cases. One may also argue, by analogy with what we have said, that the azimuth begins at the star's rising.

[6] [Nevertheless], these rules do not apply to all localities but only to some. At the equator, only the final case of the third applies, except in the case of a star that crosses the equinoctial at its rising or setting. In the first case, [i.e., if it crosses the equinoctial when rising, the azimuth point] moves along [an arc] smaller than half [a circle], by an amount equal to the occasive amplitude; and in the second case, [i.e., crossing it when setting, by an amount equal to], the ortive amplitude. If [the star] crosses [the equinoctial] between [its rising and setting] at the zenith, [the azimuth point moves] along [an arc equal to] the sum of the ortive and occasive amplitudes. [But] if [the star] crosses [the equinoctial] in the eastern quarter, [the azimuth point] moves along [an arc equal to] half plus the ortive amplitude minus the occasive amplitude. If [the star] crosses [the equinoctial] in the western quarter, [the azimuth point] moves along [an arc equal to] half plus the occasive amplitude minus the ortive amplitude.

[7] In oblique [horizons], up to a latitude close to one-eighth of the divisions [of a circle, i.e., 45°], all cases apply except the first and fifth. Similarly, at a latitude [exactly] equal to one-eighth, [all cases apply] except the first and fourth. When [the latitude] exceeds one-eighth but [is still less than] a quarter, all cases apply except the fourth. At [a latitude of a quarter], none [of the cases apply] except for the completion of the revolution in the first case, but not the intersection, since there is no initial azimuth in this [latitude].

[8] A star may move from one condition to another as it transits from one circuit to another, and if the transit [of the star] by its proper motion is considered, the first case remains unaffected. [The same holds] for cases where ortive and occasive amplitudes are taken into account, since [the transit] necessitates increases and decreases [in these amplitudes]. In the fifth and sixth cases, the difference is small and imperceptible. We have not considered the intersection [with the initial azimuth] below the horizon, since altitude is the basis for azimuth. This circle is called the "altitude circle" considering an arc from it [named the same], and is called the "azimuth [circle]" considering that, together with the initial azimuth, it encloses an arc along the horizon [named the same].

[9] This is what my limited ability allowed, and I hope it is free from errors and remedies deficiencies though the grace of the corrections of the esteemed patron, may God grant him long life.

Critical Edition of Ulugh Beg's Treatise on the Altitude Circle

The following edition is based on three manuscripts (the first two contain witnesses of Ulugh Beg's treatise; the last contains the anonymous commentary on it):

MS D: Dublin, Chester Beatty Library, Arabic 3640, ff. 143a–144a.

MS M: Tehran, Kitābkhāna-yi Madrasa-yi Marwī, 877, ff. 64b–66a.[36]

MS K: Kashan, Kitābkhāna-yi Mullā Muḥsin Fayḍ (Idāra-yi Farhang wa Irshād-i Islāmī), 27, pp. 141–171.

Among these, MS D is the most reliable. It has been corrected through collation with a copy of the commentary and heavily annotated accordingly (see Fig. 8). The scribe of MS K, in contrast, was not fully in control of the text; in some places, he appears merely to imitate his source without understanding it. Consequently, there are uncertainties in reading parts of MS K, and it is preferable to postpone its publication until additional, more reliable witnesses are discovered. Since Ulugh Beg's treatise is brief, in both manuscripts it appears within codices containing collections of short texts copied by various scribes. One of these codices (MS M) seems to have been produced and bound in Isfahan, while the other (MS D) shows evidence of having at least partially circulated in Mughal India.

The fact that only one copy of the anonymous commentary on Ulugh Beg's treatise has been discovered, and no clue of its existence is found in bio-bibliographical sources, may imply that it was not circulated widely, perhaps because its author was not a well-known scholar. But we know that at least one person used this commentary extensively to study Ulugh Beg's text. MS D is annotated extensively, with most annotations taken from the anonymous commentary. The annotations, which fill the margins and interlinear spaces, combined with an informal style of handwriting and physical damage to the pages, make parts of the text difficult to read.[37] A study of

36 Ulugh Beg's treatise in MS M starts with the following phrases:

قال السلطان الشهيد الرفيع الشأن أعجوبة العصر والزمان الغبيك بن شاهرخ بن تيمور الكوركان: بسم الله الرحمن الرحيم الحمد لله الذي رفع السماء بغير عمد، والصلوة على رسوله المسمّى بأحمد وعلى آله وأصحابه من غير عدد وعدّ.

37 I would like to express my gratitude to Dr. Moya Carey (the Curator of Islamic Collections) and Marisol Rivera (Digital Services Assistant) at the Chester Beatty Library in Dublin for providing me with the color images of the relevant pages of MS D. Without their help, deciphering the heavy annotations on these pages would be impossible.

these annotations reveals that someone who was studying Ulugh Beg's text closely added them directly to MS D. In other words, it is unlikely that they were simply transferred from another source copy of Ulugh Beg's text. At the end of several of these annotations their source is indicated, using the term *sharḥ*. In addition to this, some of the annotations are signed using the sigla *m* and *sh*. Both of these could possibly refer to the commentary.

On the other hand, due to the similarity of the text of MS D to the lammata in the commentary, one may speculate that the scribe of MS D extracted the text from a copy of the commentary rather than copying directly from an existing witness of Ulugh Beg's treatise. The person who prepared the annotations to MS D sometimes had problem in distinguishing the lemmata from the commentary, and thus he used sigla *ẓ* and *ṣḥ* for some annotations, implying that "this is apparently part of Ulugh Beg's text." Since the witness and its annotations appear to have been produced within a close temporal and cultural context, these particular annotations strengthen the possibility of MS D being derivative from the commentary.

It should be noted that there are sometimes minor variants between these annotations and the text of the commentary. In addition, there are some annotations signed with the siglum *ḥm*. In at least one of these series, the annotation is a response to what is quoted from the commentary. Such responses could be by the person who was studying Ulugh Beg's treatise or could be quoted from another witness of the text or the commentary. In another case, there are two annotations for one passage, both of them taken from the commentary but rephrased differently.

Two sets of apparatus accompany the following edition. The first records regular scribal variants among the three manuscripts. The second presents the explanatory glosses found in manuscripts D and M. As noted previously, most of the glosses in manuscript D are drawn from the anonymous commentary; therefore, we relied on manuscript K in editing these glosses. This strategy was especially helpful in cases where manuscript D is damaged. While manuscript M is generally in better condition than the other two, a full study of the transmission history of Ulugh Beg's short treatise will require examining additional witnesses that may exist in other libraries.

[١] / [د١٤٣و؛ ك١٤٣؛ م٦٤ظ] / إعلم أنّ دائرة الارتفاع / [ك١٤٤] / تنطبق على دائرة[١] نصف النهار مرّتين في دورة كوكبه، (١) لا في يوم بليلته على ما قيل، (٢) لأنّا إذا فرضنا القمر مقارنًا للشمس / [ك١٤٥] / حين وصولها إليه أوّلاً، فلا محالة[٢] في وصولها[٣] الثاني (٣) قد[٤] تمّ[٥] اليوم بليلته، والقمر ما وصل إليه،[٦] بقدر ما (٤) قطعه[٧] لسبقه[٨] (٥) في ذلك[٩] الزمان،[١٠] / [ك١٤٦] / في الكواكب التي لها طلوع وغروب، واحد (٦) فوق الأرض والآخر تحتها، وفي الابدية الظهور في أعلى ارتفاعاتها[١١] وأسفلها، إن لم يمرّ (٧) الكوكب على سمت الرأس، وإن مرّ عليه، مرّة (٨) في ارتفاعه الأسفل، (٩) أو[١٢] غاية الانخفاض،[١٣] (١٠) وفي[١٤] أعلاه على أوّل السموت، (١١) لأنّها قُبيل / [ك١٤٧] / موافاته إليه، قريب من أن تنطبق على أوّل السموت، (١٢) وإن جاز[١٥] أن يقال[١٦] (١٣) على نصف النهار (١٤) لأنّها[١٧] حينئذ[١٨] دائرة تمرّ (١٥) بسمت الرأس هي[١٩] ارتفاعية، لكن (١٦) الارتفاعية تفارق أوّل السموت حين جواز الكوكب عن سمت الرأس، فكيف يقال[٢٠] انطبقت[٢١] على نصف النهار قبل المجاوزة.

١ دائرة] –م.
٢ محالة] محة: د.
٣ فلا محالة في وصولها] وفي: م.
٤ قد] فقد: م.
٥ تمّ] مر: د. قد تمّ] قدم: ك.
٦ إليه] + بعد: د.
٧ قطعه] قطعته: د، ك.
٨ لسبقه] بسبقه: د = بسببه: ك.
٩ ذلك] ذالك: م.
١٠ الزمان] + و: شاد.
١١ ارتفاعاتها] ارتفاعها: م.
١٢ أو] هاد.
١٣ الانخفاض] الانحطاط: م.
١٤ وفي] وأمّا في: د، ك.
١٥ جاز] + حين وصوله: م.
١٦ يقال] يق: د.
١٧ لأنّها] لأنه: م.
١٨ حينئذ] + أنّه: ك، م.
١٩ هي] + دائرة: د.
٢٠ يقال] يق: د.
٢١ انطبقت] انطبق: د، ك.

[٢] وكذلك (١٧) على أوّل / [ك١٤٨] / السموت[٢٢] في الارتفاعين أو الانخفاضين المتساويين تقريبًا (١٨) إن مرّ الكوكب عن سمت الرأس[٢٣] إلى جانب[٢٤] القطب الخفي إلى أن يماسّ مداره سمت القدم، ومرّة[٢٥] إن مرّ الكوكب[٢٦] عليها. / [ك١٤٩] / ولا ينطبق عليه إن مرّ عن[٢٧] سمت[٢٨] الرأس إلى[٢٩] الظاهر، (١٩) ما هو إلى عرض البلد، (٢٠) أو إلى الخفي ما هو بخلافه.

[٣] ونقطة تقاطع الارتفاعية مع الأفق يقال[٣٠] لها نقطة السمت، وإنّما عبّرنا[٣١] (٢١) بهذا[٣٢] (٢٢) لأنّ السمت ينعدم في دورة مرّتين، / [ك١٥٠] / تدور على الأفق[٣٣] / [م٦٥و] / بحسب وقوع المدارات. (٢٣) والمدار إمّا أن[٣٤] يقطع أوّل[٣٥] السموت فقط، / [د١٤٣ظ] / أو مع الأفق،[٣٦] أو لا يقطع[٣٧] أوّل[٣٨] السموت، ومنه (٢٤) في[٣٩] الخفي، (٢٥) أو يماسّه ويقطع الأفق، أو يماسّه ولا يقطع الأفق،[٤٠] أو لا يلاقيه (٢٦) سواء كان قاطعًا للأفق[٤١] أو لا. والمدار على[٤٢] الثلاثة[٤٣] الأخيرة[٤٤] يكون في جانب[٤٥] الظاهر، (٢٧) / [ك١٥١] / فالأقسام ستّة.

٢٢ على أوّل السموت] –د.
٢٣ فكيف قال ... سمت الرأس] هام.
٢٤ جانب] –م.
٢٥ مرة] + واحدة: د (+«شرح» فوق السطر).
٢٦ الكوكب] –م.
٢٧ عن] على: ك.
٢٨ سمت] السمت: ك.
٢٩ عن سمت الرأس إلى] فحمل: م.
٣٠ يقال] يق: د.
٣١ عبّرنا] + (مطموس): شاد.
٣٢ بهذا] لهذا: ك.
٣٣ على الأفق] –د.
٣٤ أن] –ك.
٣٥ أوّل] + السموت: شام.
٣٦ الأفق] + أو يماسّه ولا يقطعه أو لا يقطع الأفق: شام.
٣٧ يقطع] + الأفق أو لا يقطع: م.
٣٨ أوّل] + السموت: د.
٣٩ في] + جانب: م.
٤٠ الأفق] شاد.
٤١ للأفق] تاد = الأفق: ك.
٤٢ والمدار على] وعلى: م.
٤٣ الثلاثة] الثلث: م.
٤٤ الأخيرة] + (مطموس): شاد.
٤٥ جانب] هاد.

[٤] تتمّ[٤٦] الدورة على الأوّل. (٢٨) / [ك١٥٢] / وتدور على قوس[٤٧] أكثر من النصف بقدر مجموع سعة المشرق وسعة المغرب[٤٨] للكوكب على الثاني. / [ك١٥٤] / وعلى النصف (٢٩) أو على أقلّ منه (٣٠) بقدر مجموعهما على الثالث. / [ك١٥٥] / وعلى قوسين كلّ منهما أقلّ من الربع بقدر تماميهما[٤٩] (٣١) على الرابع. / [ك١٥٦] / وعلى النصف على الخامس. / [ك١٥٧] / وتبتدي من المغرب[٥٠] (٣٢) وتتردّد[٥١] (٣٣) على أقلّ من النصف / [ك١٥٩] / بقدر ما يقتضي نصفها (٣٤) المدار، (٣٥) ويحصل جيب ذلك[٥٢] (٣٦) من قسمة جيب تمام البعد على جيب تمام عرض البلد منحطًا،[٥٣] (٣٧) / [ك١٦١] / إن لم[٥٤] يقطع الأفق، وإن قطعه فلا،[٥٥] (٣٨) بل[٥٦] يتوقّف التردّد[٥٧] أواسط القوس المذكورة،[٥٨] (٣٩) / [ك١٦٢] / على السادس. (٤٠)

[٥] والكوكب لا ينتقل (٤١) من الربع الذي كان[٥٩] عليه من الدائرة[٦٠] الارتفاعية[٦١] إلى الآخر.[٦٢] (٤٢) ويمكن أن يقال[٦٣] لا تتمّ الدور في الأوّل، (٤٣) / [ك١٦٣] / ولا تدور على ما وصفنا في الخامس والسادس[٦٤] عند نقطة منسوبة (٤٤) إلى ما نسب إليه القطب[٦٥] الظاهر، وهي

٤٦ تتمّ] هاد.
٤٧ قوس] –م.
٤٨ وسعة المغرب] والمغرب: م.
٤٩ تماميهما] تمامها: م = تمامي سعة المشرق والمغرب: ك.
٥٠ وتبتدي من المغرب] –م.
٥١ وتردّد] وتتردّد: ك.
٥٢ ذلك] ذالك: م.
٥٣ عرض البلد منحطًا] العرض: د (+«منحطًا» في الهامش)، ك.
٥٤ لم] + يقطع: شام.
٥٥ فلا] فاد.
٥٦ فلا بل] –م.
٥٧ التردّد] + في: م.
٥٨ المذكورة] –م.
٥٩ كان] –م.
٦٠ الدائرة] –م.
٦١ الارتفاعية] + إلّا على الرابع والخامس بعد الموافاة من أحدهما: م.
٦٢ الآخر] الأخرى: م.
٦٣ يقال] يق: د.
٦٤ في الخامس والسادس] شاد.
٦٥ القطب] –م.

منتصف (٤٥) ما تدور عليه على الأخيرين، (٤٦) لانعدام الارتفاع. (٤٧) ثمّ[٦٦] إن صحّ هذا / [ك١٦٤] / تدور على الربع الشرقي وكذلك[٦٧] على[٦٨] الغربي (٤٨) والجنوبي[٦٩] (٤٩) وعلى قوسين (٥٠) كذلك (٥١) على الخامس وعلى السادس.[٧٠] / [ك١٦٥] / ويمكن أن يقال،[٧١] (٥٢) قياسًا على ما نحن[٧٢] نقول، ابتداء السمت[٧٣] من عند الطلوع. (٥٣)

[٦] وهذه[٧٤] (٥٤) الأحكام[٧٥] لا تجري في الآفاق (٥٥) كلّها (٥٦) بل تجري في بعضها البعض. وفي[٧٦] الاستوائية / [م٦٥ظ] / لا يوجد (٥٧) إلّا أخير (٥٨) الثالث، (٥٩) إلّا في كوكب يقطع المعدّل حين طلوعه أو غروبه، تدور[٧٧] على أقلّ من النصف بقدر سعة المغرب[٧٨] / [ك١٦٦] / على الأوّل، (٦٠) وسعة[٧٩] المشرق على الثاني، وعلى مجموع سعة المشرق والمغرب إن يقطعه بينهما على سمت الرأس، وتدور على[٨٠] مجموع النصف وسعة المشرق إلّا سعة المغرب إن قطعه في الربع الشرقي، / [ك١٦٧] / وتدور على[٨١] مجموع النصف وسعة المغرب إلّا[٨٢] سعة المشرق إن قطعه في الربع الغربي.

٦٦ ثمّ] –م.
٦٧ وكذلك] وكذالك: م.
٦٨ وكذلك على] شاد.
٦٩ والجنوبي] شاد.
٧٠ على الخامس وعلى السادس] على السادس: د. والجنوبي وعلى قوسين كذلك على الخامس وعلى السادس] على الخامس وعلى أقلّ من الربع كذلك على السادس: م.
٧١ يقال] يق: د.
٧٢ نحن] –م.
٧٣ السمت] –م.
٧٤ وهذه] شاد.
٧٥ الأحكام] –د.
٧٦ وفي] + الآفاق: شاد.
٧٧ تدور] –م.
٧٨ المغرب] المشرق: د (+«المغرب ظ» تحت السطر)، ك.
٧٩ وسعة] وبقدر سعة: د.
٨٠ وتدور على] وعلى: م.
٨١ وتدور على] وعلى: م.
٨٢ إلّا] على: شام (+«إلّا» في الهامش).

[٧] وفي المائلة إلى عرض قريب من الثمن الأقسام، كلّها موجودة إلّا الأوّل والخامس، / [ك١٦٨] / وكذلك[٨٣] في عرض يساوي الثمن إلّا الأوّل والرابع، وإن جاوز[٨٤] / [د١٤٤و] / الثمن يوجد فيه[٨٥] كلّ الأقسام إلى عرض يساوي الربع إلّا الرابع. وفيه ليس شيء[٨٦] إلاّ الأوّل من تتميم الدورة لا من[٨٧] القطع لأنّ[٨٨] (٦١) ليس فيه أوّل السموت.

[٨] والكوكب يتحوّل[٨٩] من كلّ[٩٠] قسم (٦٢) إلى[٩١] قسم بحسب انتقاله / [ك١٦٩] / من مدار إلى مدار، وإن لوحظ (٦٣) انتقاله بخاصّته لا يخلّ[٩٢] الأوّل، وفيها تلاحظ فيه سعة المشرق والمغرب، لأنّ[٩٣] يقتضي[٩٤] الزيادة والنقصان.[٩٥] (٦٤) / [ك١٧٠] / وفي الخامس والسادس[٩٦] (٦٥) يكون تفاوت قليل لا يحسّ به. وما (٦٦) اعتبرنا قطعه إيّاه تحت الأرض لأنّ منشاء[٩٧] السمت الارتفاع. وهذه الدائرة بملاحظة قوس منها[٩٨] تسمّى الدائرة[٩٩] الارتفاعية، وبملاحظة حصرها قوسًا من الأفق مع أوّل / [م٦٦و] / السموت[١٠٠] السمتية.

[٩] وهذا ما سمحت به قريحة / [ك١٧١] / وارجو أن يعري من الخلل ويشفي من العلل بيمن إصلاح المخدومية، (٦٧) متّعنا الله بطول بقائه.[١٠١] (٦٨)

٨٣ وكذلك] وكذالک: م.
٨٤ جاوز] +عرض: م.
٨٥ فيه] –م.
٨٦ شيء] –م.
٨٧ من] –د.
٨٨ لأنّ] لأنّه: م.
٨٩ يتحوّل] يتجوّز: م.
٩٠ كلّ] –م.
٩١ إلى] (مطموس): شاد.
٩٢ يخلّ] +في قولنا في: م.
٩٣ لأنّ] لأنّه أن: م.
٩٤ يقتضي] اقتضي: م.
٩٥ والنقصان] أو النقصان يكون هو هما: م.
٩٦ الخامس والسادس] السادس والخامس: م.
٩٧ لأنّ منشاء] لا يشار: م (+«لأنّ منشاء السمت الارتفاع» مع رمز «ظ ر»).
٩٨ منها] منه: م.
٩٩ الدائرة] –م.
١٠٠ السموت] (مطموس): شاد.
١٠١ بقائه] + تمّت: د. قريحة وارجو ... بقائه] تمّت الرسالة الملكية: م.

[الحواشي]

(١) كوكبه] + ضمير كوكبه عائد إلى نصف النهار والإضافة الأولى ملابسة شرح + إنّي أرى أنّ الضمير عايد إلى الارتفاع حم: هاد. (٢) قيل] + چغميني: فاد. (٣) الثاني] + شرح: فاد. (٤) بقدر ما] + مطالع: تاد. (٥) لسبقه] + أي بسبب أنّه سبق القمر على الشمس في حركة الغربية شرح: هاد. (٦) واحد] + أي مرّة واحدة الأولى واحدة ففي (مطموس) الأخرى: تام. (٧) إن لم يمرّ] + والانطباق المذكور على الوجه المسطور من كلا القسمين ليس على إطلاقه فلذا قال إن لم يمرّ الخ: هاد. (٨) الأسفل] + ينطبق على نصف النهار: تام. (٩) الأسفل] + إن كان الكوكب أبدي الظهور ش: تاد. (١٠) الانخفاض] + إن كان له طلوع وغروب ش: فاد. (١١) على أوّل السموت] + أي ينطبق دائرة ارتفاعه عليه: تام. (١٢) السموت] + ويبعد عن نصف النهار آنًا فآنًا: تام. (١٤) النهار] + يصدق عليه ظ: فام. (١٣) يقال] + أي يطلق شرح: تاد. (١٥) تمرّ] + ظ ونقطة مفروضة هي مركز الكوكب صحّ: هاد. (١٦) لكن] + هذا تأكيد للسابق: تام. (١٧) وكذلك] + أي وكذلك تنطبق على أوّل السموت شرح: فاد. (١٨) تقريبًا] + إنّ الكوكب إذا وصل إلى الارتفاع الشرقي أو الانحطاط الشرقي يكون على مدار ثمّ إذا وصل إلى الآخر انتقل إلى مدار آخر لا محالة يكون ارتفاع تقاطعه مع أوّل السموت أو انحطاطه أقلّ أو أكثر من الأوّل: تاد. (١٩) إلى الظاهر] + أي إلى القطب الظاهر: فاد. (٢٠) البلد] + يحتمل أن يكون ما هو إلى عرض البلد بدلًا عنه بدل الكلّ أو مفسّرًا له أو منصوبًا على المدح شرح: هاد. (٢١) عبّرنا] + أي عن نقطة السمت شرح: هاد. (٢٢) بهذا] + أي بقولنا نقطة تقاطع الارتفاعية مع الأفق مع المشهور فيما بين الجمهور أنّ الشرق غير (مطموس) من المشرق (مطموس) من قوس السمت (مطموس): هاد. (٢٣) المدارات] + فعلى المشهور عند الجمهور لا يطلق على مطلع الاعتدالي ومغيب الاعتدالي: هاد. (٢٤) ومنه] + أي من أوّل السموت ش: فاد. (٢٥) الخفي] + أي من الجانب القطب الخفي ش: فاد. (٢٦) يلاقيه] + أي أوّل السموت: تاد. (٢٧) الظاهر] + أي القطب الظاهر: هاد. (٢٨) الأوّل] + يعني أنّ نقطة السمت تنتقل على الأفق وتتمّ دورته على الأوّل لأنّ الكوكب إذا كان على ارتفاعه الأسفل يكون دائرة ارتفاعه ينطبقه على نصف النهار فيكون نطقتا السمت متحدتين بنقطتي الشمال والجنوب وبحسب ازدياد الارتفاع يزداد

تباعد نقطتي السمت عنهما إلى أن يصل أن: هام. (٢٩) النصف] + إذا كان الكوكب على مدار يماسّ لمعدّل النهار م: تاد. (٣٠) أقلّ منه] + إذا كان الكوكب على مدار غير مماسّ بمعدّل النهار (مطموس): فاد. (٣١) تماميهما] + أي مجموع سعة المشرق والمغرب: تاد. (٣٢) المغرب] + أي من جانب المغرب إلى نقطة المغرب: تاد. (٣٣) وتتردّد] + أي تدور على أقلّ من النصف ثمّ ترجع عليه: فاد. (٣٤) يقتضي نصفها] + تقييد الاقتضاء بالنصف لا يكون احترازًا عن شيء بل هو بيان واقع إذ من البيّن أنّ المدار لمّا كان مقتضيًا لتلك القوس الدائر عليها نقطة التقاطع فبالضرورة يكون مقتضيًا لنصفها أيضًا شرح: هاد. (٣٥) المدار] + وتوضيح ذلك أنّه إذا تماسّ على هذا التقدير دائرة الارتفاع مدار الكوكب في جهة الشرق مثلًا حصل مثلّث أحد أضلاعه قوس من هذه الدائرة من سمت الرأس إلى الأفق والآخر قوس ربع نصف النهار والثالث من الأفق بين الشمال مثلًا والسمت وهذا قاعدة ذلك المثلّث المذكور. ثمّ إذا توهّمنا دائرة ميل تارة بنقطة التماس المذكورة حدث مثلّث أصغر أحد أضلاعه قوس تمام ارتفاع نقطة التماسّ والآخر تمام عرض البلد وقاعدته تمام البعد. ونسبة قاعدة الأصغر أعني تمام البعد إلى قاعدة الأكبر أعني المجهول كنسبة تمام عرض البلد إلى الربع. فبالأربعة المتناسبة إذا قسم تمام البعد على تمام عرض البلد منحطًا حصلت قاعدة المثلّث الأكبر حم: هاد. (٣٦) ذلك] + النصف: تاد. (٣٧) منحطًا] + أي عرض البلد: تاد. + أي حال كون تمام عرض البلد منحطًا: هاد. (٣٨) فلا] + أي فلا يتردّد على مجموع تلك القوس: تاد. (٣٩) المذكورة] + ويتكرّر في طرفيها: تاد. (٤٠) السادس] + أي كلا الحكمتين المذكورتين على السادسة: هاد. (٤١) ينتقل] + على جميع الأقسام: تاد. (٤٢) الآخر] + إلّا على الذي يصل الكوكب إلى سمت الرأس فإنّ الكوكب ينتقل بعد الوصول من أحدهما إلى الآخر صحّ ظ: هاد = + أعني على سمت الرأس شرح: هاد = + أي على القسم الذي يصل الكوكب إلى سمت الرأس وهو الرابع والخامس ش: هاد = + إلّا على الرابع والخامس بعد الموافاة من أحدهما: م. (٤٣) الأوّل] + لأنّ أحد قسميه أن يكون مدار الكوكب مماسًا للأفق ولا شكّ أنّ الكوكب إذا كان على التماس ليس له ارتفاع بل ابتداء نقطة تقاطع الارتفاعيه من نقطة بل نقطة التماس بعيد جواز الكوكب عنها وانتهاءها (مطموس): هاد. (٤٤) منسوبة] + أعني نقطة الشمال في الشمالي والجنوب في الجنوبي: تاد. (٤٥) منتصف] + أي منتصف قوس تدور نقطة السمت على تلك القوس: تاد. (٤٦) الأخيرين] + أعني الخامس والسادس: فاد. (٤٧) الارتفاع] + في قسم من أقسام كلّ منهما م: فاد. (٤٨) الغربي] + على النصف الشرقي وكذلك(مشطوب) على(مشطوب)

الغربي صحّ: هاد. (٤٩) والجنوبي] + على الخامس صحّ: تاد. (٥٠) قوسين] + كلّ منهما أقلّ من الربع صحّ: تاد. (٥١) كذلك] + أي كما تدور تلك النقطة على الخامس على الربع الشرقي والغربي: تاد. (٥٢) يقال] + تتمّ وتدور صحّ: تاد. (٥٣) الطلوع] + بأن يقال ابتداء نقطة تقاطع الارتفاعية فيها من عند النقطة المنسوبة قياسًا على ما نحن الخ: هاد. (٥٤) وهذه] + أي هذه الأحكام م: تاد. (٥٥) الآفاق] + أي في كلّ من الآفاق: تاد. (٥٦) كلّها] + أي كلّ الأحكام: تاد. (٥٧) لا يوجد] + (مطموس) موجود في الاستوائية فقط حم: تاد. (٥٨) أخير] + وهو أن تدور نقطة السمت على أقلّ من نصف الأفق بقدر مجموع سعة المشرق والمغرب: تاد. (٥٩) الثالث] + مثلًا إذا طلع الكوكب من شمال المشرق حين كونه (مطموس) بعده جزء (مطموس) (مطموس) في التقارب إلى نقطة المشرق إلى أن يصل الكوكب إلى المعدّل على سمت الرأس فتنطبق على المشرق وقطع سعة المشرق ثمّ إذا جاوز الكوكب عن المعدّل وشرع في تباعد عنه ابتدأت تلك النقطة من المغرب (مطموس) أخذت في التباعد عنها إلى أن يغرب الكوكب فحينئذ قطعت سعة المغرب شرح: هاد. (٦٠) الأوّل] + والأوّل أيضًا لا يجري فيه بجميع الأحكام بل ببعضها فلهذا قال من تتميم الدورة صحّ: هاد. (٦١) لأنّ] + هي مخففة من المثقلة حذف إسمها الذي هو ضمير الشأن قال ابن الحاجب وحذفه منصوبًا ضعيف إلّا مع أنْ إذا خفّفت فإنّه لازم نحو قوله تعالى ﴿عَلِمَ أَنْ سَيَكُونُ﴾ شرح: تاد. (٦٢) قسم] + من الأقسام الستّة: هاد. (٦٣) وإن لوحظ] + أي من غير أن يلاحظ التحوّل من قسم إلى آخر بحسب الانتقال من مدار إلى آخر فتلك الملاحظة لا يخلّ في الحكم المذكور في القسم الأوّل وهو تتميم الدورة وكذلك فيما يلاحظ فيه سعة المشرق والمغرب وهو القسم الثاني والثالث والرابع لأنّه يقتضي تلك الملاحظة الزيادة والنقصان في سعة المشرق والمغرب: هاد. (٦٤) الزيادة والنقصان] + أي سعة المشرق والمغرب بعد الزيادة والنقصان مبتداء وتذكيره باعتبار هذا اللفظ وهو مبنيًا وخبره هما أى سعة المشرق والمغرب اللتين ذكرناهما في القسم الثاني والثالث لأنّهما يقبلان الزيادة والنقصان ويكون الجملة في محل النصب خبر يكون وإسمه ضمير الشأن فاندفع ما توهّم أنّه ينبغي أن يقال يكون هو إيّاهما لوجوب نصب خبر كان فإن قيل ينبغي أن يقال يكون هو هو لوجوب المطابقة بين المبتداء والخبر قلنا إنّما قيل هكذا النكتة وهي التنبيه على أنّ مرجع المبتداء عند التحقيق شيئان شرح: هاد. (٦٥) وفي الخامس والسادس] + أي فيهما تلك الملاحظة وإن كانت محله لكن يحصل تفاوت قليل لا يحسّ به فتأمّل فيهما شرح: هاد. (٦٦) وما] + نافية: تاد. (٦٧) المخدومية] + أعني موسى المشتهر بقاضي زادة رومي:

تاد. (٦٨) بقائه] + قوله **بقدر ما يقتضي نصفها المدار** الخ بيان القوس المتردّد عليها لا للتفاوت على قياس قوله بقدر مجموعها وبقدر تمامها كما هو المتبادر يعني أنّ القوس المتردّد عليها بقدر تقتضيها المدار ويحصل جيب نصفها من القسمة المذكورة إلّا إذ اخترنا قوسًا بين قطب المعدّل ونقطة تماسّ الارتفاعية والمدار يحدث مثلّثان مشتركان في زاوية على سمت الرأس أحد أضلاع المثلّث الأعظم من دائرة نصف النهار وهو ربع والآخر من الارتفاعية وهو أيضًا ربع والثالث من الأفق وأحد أضلاع المثلّث الأصغر من نصف النهار وهو تمام عرض البلد والآخر من الارتفاعية وهو من سمت الرأس إلى نقطة التماسّ الثالث من نقطة التماس إلى القطب وهو تمام بعد الكوكب نسبة تمام عرض البلد من المثلّث الأصغر إلى نظيره من المثلّث الأعظم بل نسبة جيبه إلى جيبه كنسبة جيب تمام البعد من الأصغر إلى جيب قطره من الأعظم على ما يعلم من المجسطي فيحصل الرابع بالضرب والقسمة إلّا أنّه اقتصر على القسمة منحطًا وقد عرفت معناها في فصل جعل ما ينحل في أواخر القول الخامس: هام.

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